When Demand exceeds Capacity it hurts Supply Chains

https://www.learnaboutlogistics.com/when-demand-exceeds-capacity-it-hurts-supply-chains/

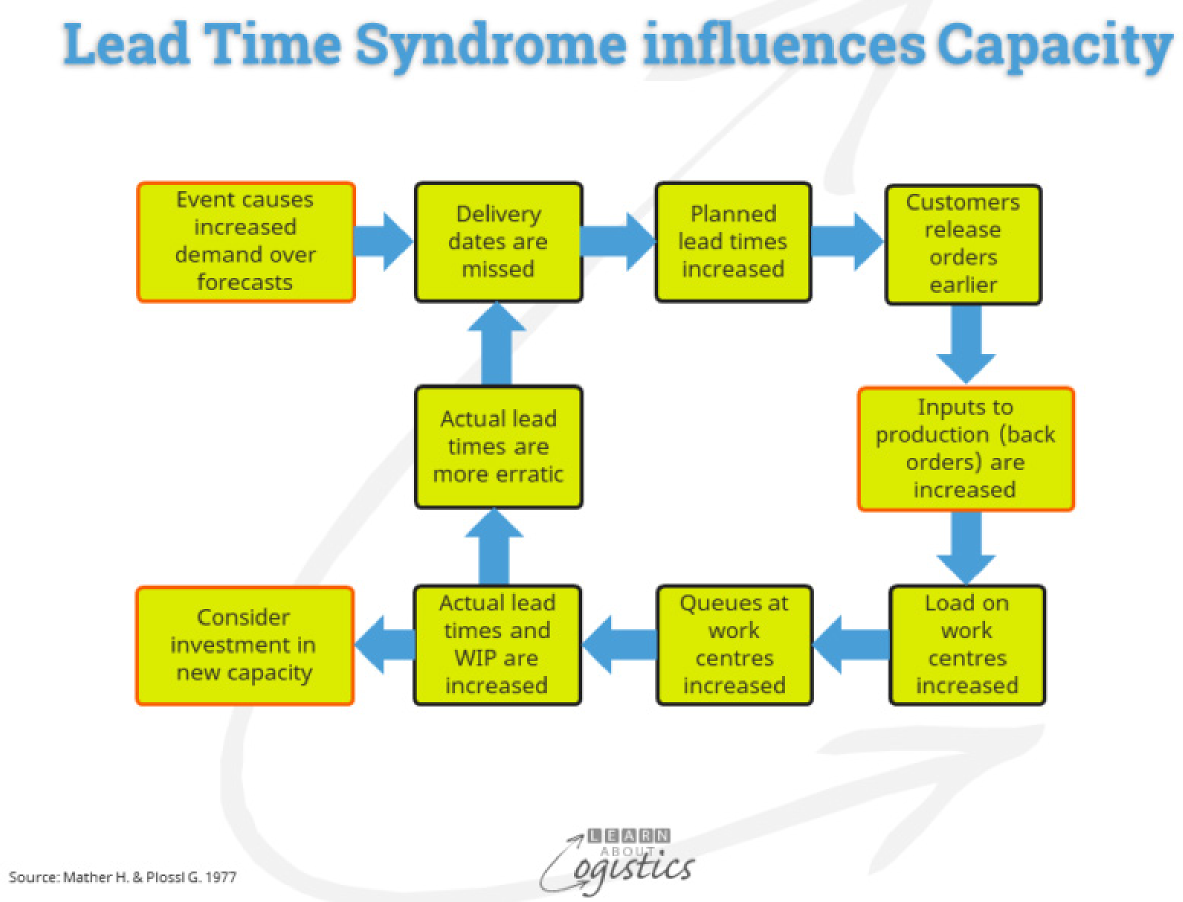
Roger Oakden May 10, 2021

Lead Time Syndrome

The flexibility of a supply chain or an operation is based on the time to change (or flex) capacity and the total cost to change. Automotive assembly lines may have a flex time of up to six months to change the output of cars per day and with a substantial cost.

The aim of Operations Planning (also called production planning and control) is to meet the Sales & Operations Planning (S&OP) targets concerning: reliability of due dates; shortest lead times; high capacity utilization and low work in progress (WIP) levels, while maintaining the productivity targets of the organisation.

An unexpected event like COVID 19 can increase the order load on a facility and the options available to accommodate new customer orders are reduced. Planners may increase the lead time and/or split orders into transfer batches to get some of the orders through the facility. Increasing lead times has risks and splitting orders may require more downtime for equipment changeovers, so capacity is reduced.



Lead Time Syndrome influences Capacity

Source: Mather H. & Plossl G. 1977

The effect of increasing lead times can cause a business to experience a cycle of missed delivery dates, as shown in the diagram. As delivery dates are missed and planners increase the planned lead times, customers are likely to order more items to cover the increased lead time. Back-orders will increase, as erratic ordering behaviour results in even larger variability.

A continual increase in lead times and back-orders makes this situation worse. Directors of the supplier business may even authorise construction of new facilities. Critically, when a new facility commences operations, capacity is increased and therefore the lead time for new orders is reduced. Consequently, as customers have already placed orders to cover the extended lead time, they have no need to place new orders. The supplier will progressively reduce the lead time to attract orders until the lead time falls to the original level. At worse, the new facility is then closed due the lack of orders – real demand has not substantially changed!

This outcome provides an example of the need for Supply Chain professionals to understand the interconnectedness of all elements through supply chains. In the situation of increased lead times, the interaction of variables, such as: capacities, planned and actual lead times, suppliers' lead times, work in process levels etc., can lead to unforeseen problems when trying to meet the achievement targets.

The situation described is not hypothetical; it has occurred on more than one occasion, with severe consequences for the affected businesses and their people. In

1977, the process was called the Lead Time Syndrome by Hal Mather and George Plossl, following experience of the US semiconductor industry. It appears that 45 years later, the semiconductor industry is repeating the same cycle.

Semiconductor supply chains

A previous blogpost discussed the shortage of semiconductors for the automotive industry. Since then, it has been reported that semiconductor shortages will affect the availability of most products that rely on electronics. This is the Lead Time Syndrome in action.

Automotive semiconductors are produced on older generation machines which are not interchangeable with later model machines, so capacity is relatively fixed. Total demand from the automotive industry is less than 10 percent of the semiconductor suppliers' total sales. Hopefully, it is viewed as a separate market segment; however, the input materials are similar.

The potential shortage of semiconductors was first identified in Q2 2020, due to shortages of silicon wafers from the foundries, substrate materials and other discrete components, caused by COVID 19 shutdowns. However, the extent of the problem only came to light in Q4.

The suppliers of semiconductors are Tier 2 or 3 suppliers in an automotive or electronic supply chain, engaged in Assemble, Test and Package (ATP). They supply to Tier 2 component suppliers or Tier 1 suppliers of sub-assemblies. Upstream from semiconductor suppliers are the 'foundries', where silicon wafers have circuits etched, then the wafers are cut into chips for ATP. The suppliers at each tier have their suppliers of designs, materials and equipment, which provides for complex and global supply chains for brand companies.

When demand for computer chips was increasing, due to 'work from home' directives under COVID 19, the supply of materials was allocated to consumer electronics, given the low level of sales to automotive. When it became evident that the output of vehicles would be limited due to the shortage of semiconductors, brand companies in other sectors took action in their markets by delaying the launch of new products and allocating products to retailers. They also influenced their Tier 1 and 2 suppliers to increase orders for semiconductors – Just in Case!

In normal times, the lead time for a semiconductor order is more than three months. As capacity remains the same, lead times have been increased and the delays will affect the output of finished products. If the orders for semiconductors continue to increase as a reflection of increased lead times, the availability of consumer finished products will be affected until there is a 'circuit-breaker' in demand or supply.

For demand, there could be a reduction in consumer interest, as they experience long lead times for deliveries and the inability to negotiate prices or terms. For supply, capacity can be increased through building new foundries and factories, which has commenced with announcements of potential investments in the US and

Europe. New capacity could therefore be available in late 2022 onwards, which means another18 months of disruption. However, the effect will only be known after the additional capacity is made available – has demand really increased, or will new capacity remain idle?

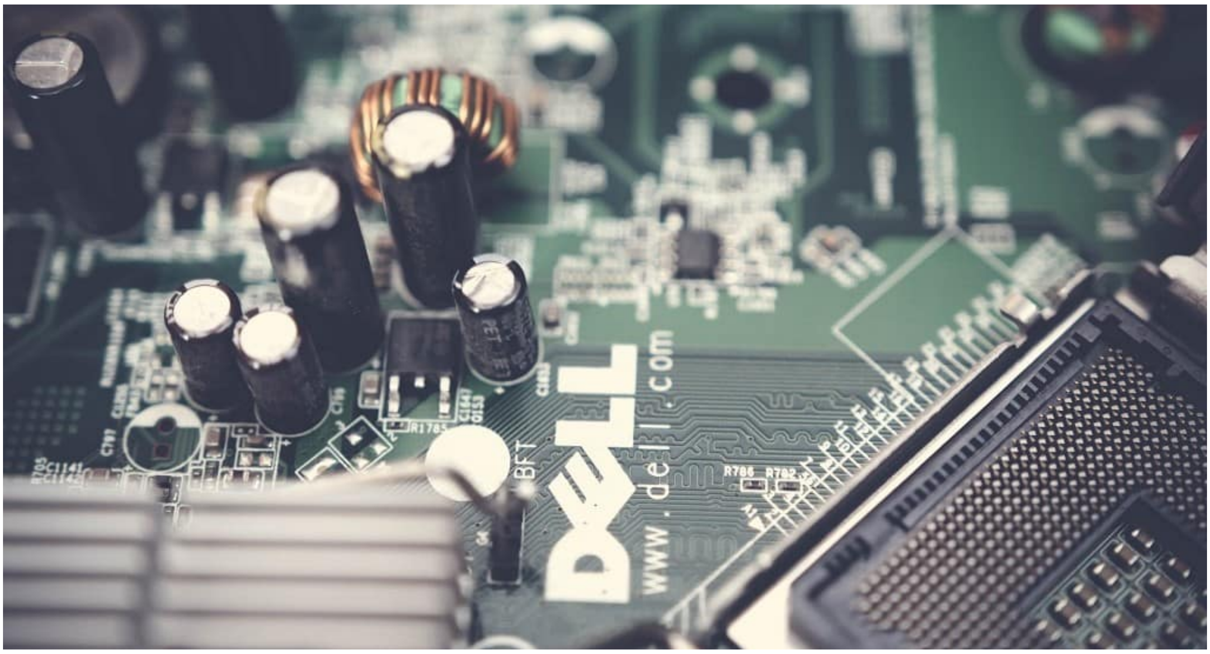
Addressing the Lead Time Syndrome (LTS)

While the LTS problem can be described, a systems based 'solution' is currently not available, but actions can be taken to ensure that lead times can be met. The three ways to enable lead-times for customers is through available capacity, inventory or reducing time in Operations. To reduce throughput time consider:

- The *Theory of Constraints* enables planners to identify bottlenecks in operations
- Within each planning slot or bucket the total time (or load) consists of:
    - Productive work – the processing time
    - Non-productive work, including time for set-up, cleaning and waiting. Single minute exchange of dies (SMED) should be the aim for set-ups. Wait time (for resources to be available) should be another focus for improvement. Manage the overall equipment efficiency (OEE) to have the productive capacity available

To reduce the impact of demand fluctuations, supply chain professionals need to understand their supply network. It is a complex, adaptive systems (CAS) which cannot be 'managed', but only analysed to enable a better understanding for possible actions to be taken.

Push-Pull Supply Chain Strategy – Dell



There was a time, not too long ago, when Dell dominated the market for personal computers (PCs). It was a time that saw the company spawn a whole new industry of direct buying. Dell focused on a strategy that bypassed distribution channels by avoiding retailers and wholesalers altogether.

Instead of dealing with the added costs of distribution, Dell made the decision to sell directly to consumers. They then added to their value assertion by providing unmatched 24/7 technical support, shortened delivery times and unbeatable warranties.

The idea was to provide direct customer support, limit inventory levels and provide a relatively inexpensive customised finished good. By avoiding distribution, Dell was able to offer unmatched pricing to end-users for customised PCs.

It wasn't long before other companies would try to copy Dell's strategy. However, many companies soon learned that Dell's approach combined a direct sales model with a push-pull procurement and supply chain strategy, one where Dell reduces its lead times on customised offerings by procuring exactly what's needed to complete a customer's order.

Dell's Direct Sales and Supply Chain Strategy

In many ways, Dell's process borrows from "Just in Time" (JIT) supply chains in terms of limiting inventory levels. However, Dell focuses more on a push-pull strategy, one where it pushes options to its customers and then uses that customer order to pull demand through Dell's supply chain. At least, that's how Dell started their process.

Nowadays, it seems as if Dell has somewhat abandoned that strategy altogether with respect to offering customised PCs as those options are no longer as vital as they once were. However, that doesn't mean that Dell's strategy is any less important in today's marketplace. In fact, there are several companies that can run a variation of Dell's approach.

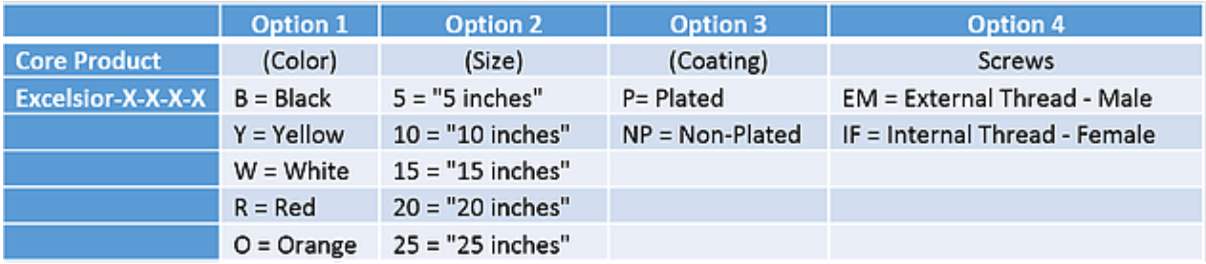
An Example of a Push-Pull Strategy

Instead of manufacturing the same product from a fixed materials list, a push-pull strategy combines a fixed materials list with a flexible materials list, one where the flexibility is tied into the options customers choose.

However, companies running a push-pull strategy don't tend to let customers choose just any option. Instead, they control the options that are provided to customers so as to minimise lead times on delivery. In essence, the approach is to "push" customers to choose specific options. Once those options are chosen, the customer's order "pulls" demand through the company's supply chain.

Another benefit of the strategy is that the nomenclature for the product being purchased is directly tied to the options chosen by a given customer. For instance, the first few numbers and or letters of the core product are fixed while the individual options that follow are not.

Here is an example of what the ordering process may look like for a company offering a customised finished good through a push-pull process.



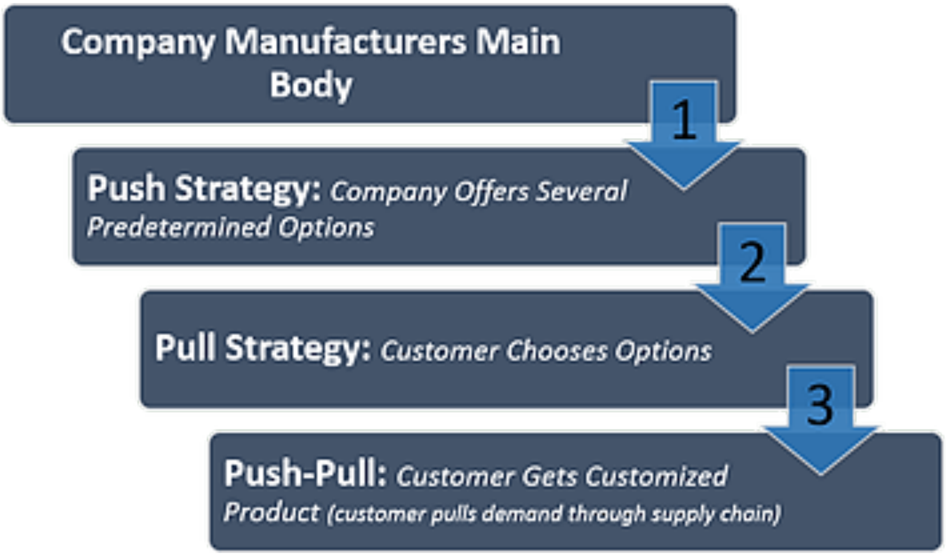
	Option 1	Option 2	Option 3	Option 4
Core Product	(Color)	(Size)	(Coating)	Screws
Excelsior-X-X-X-X	B = Black	5 = "5 inches"	P= Plated	EM = External Thread - Male
	Y = Yellow	10 = "10 inches"	NP = Non-Plated	IF = Internal Thread - Female
	W = White	15 = "15 inches"		
	R = Red	20 = "20 inches"		
	O = Orange	25 = "25 inches"		

The core product (Excelsior) stays fixed and is made from a fixed materials list. It’s the main body of the product and we can safely assume it is sitting on the shelf in the company’s warehouse. The end product is considered to be in a semi-finished state waiting for customers to place orders for their chosen options.

So, in this case, the Excelsior “body” is waiting to be finished with the customer-specific options. The list of options that follow (Option 1, 2, 3 and 4) allow the customer to choose a customised solution with minimal delivery times. For the sake of simplicity, we’ve kept the options fairly straightforward. However, it’s not uncommon for companies to have a long list of options.

Let’s assume a customer orders the Excelsior-B-25-P-IF product offering. This means they’ve ordered an Excelsior product that will be painted black, 25 inches long, plated, and have internal threads/female screws.

However, the company has all these options ready the moment the customer places the order because they have a demand history to call upon. In this case, the company has manufactured most of the finished good and is simply waiting for customers to place orders. When those customers order, they get the options they want and immediate delivery.



In order for the strategy to work, the company must manufacture or hold the core product on its shelf. Or, at the very least, it must have enough to quickly assemble the core product in order to maintain the time-critical delivery for the customer. In many ways, it’s as if the company pre-assembles a vast majority of the finished good and then waits for the customer to place their specific order.

Some businesses see the push-pull strategy as a combination of two distinct sales and/or marketing strategies. They may come to see the push strategy as relying upon multiple sales channels who push products to customers.

This might include selling through distribution or retail channels by offering those channels discounts, stocking deals and or rebates in order to incentivise sales. They then see the pull strategy as being separate and focused entirely on the end-user.

Dell’s strategy is unique in that it uses the savings accrued by avoiding distribution to offer unbeatable prices to the everyday consumer. It focuses directly on the end-user by bypassing those aforementioned sales channels and then passes on those savings with lower prices.

However, Dell also understands the benefit that those distribution sales channels provide in terms of technical support and warranties. So, it offers those same services to its end-user customers and at a much lower cost than what those distribution channels could offer.

In the end, Dell’s strategy was simple and straightforward. It combined a customer order strategy that drove its supply chain. The impetus was on driving customer demand and then using that customer demand to better manage procurement and inventory. It reduced pricing, secured high-volume business and helped to position Dell as a market leader.

https://www.paultrudgian.co.uk/push-pull-supply-chain-strategy/

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The Bullwhip Effect in Supply Chains

SPRING 1997

Distorted information from one end of a supply chain to the other can lead to tremendous inefficiencies: excessive inventory investment, poor customer service, lost revenues, misguided capacity plans, ineffective transportation, and missed production schedules. How do exaggerated order swings occur? What can companies do to mitigate them?

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The Bullwhip Effect in Supply Chains

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Distorted information from one end of a supply chain to the other can lead to tremendous inefficiencies: excessive inventory investment, poor customer service, lost revenues, misguided capacity plans, ineffective transportation, and missed production schedules. How do exaggerated order swings occur? What can companies do to mitigate them?

Not long ago, logistics executives at Procter & Gamble (P&G) examined the order patterns for one of their best-selling products, Pampers. Its sales at retail stores were fluctuating, but the variabilities were certainly not excessive. However, as they examined the distributors' orders, the executives were surprised by the degree of variability. When they looked at P&G's orders of materials to their suppliers, such as 3M, they discovered that the swings were even greater. At first glance, the variabilities did not make sense. While the consumers, in this case, the babies, consumed diapers at a steady rate, the demand order variabilities in the supply chain were amplified as they moved up the supply chain. P&G called this phenomenon the "bullwhip" effect. (In some industries, it is known as the "whiplash" or the "whipsaw" effect.)

When Hewlett-Packard (HP) executives examined the sales of one of its printers at a major reseller, they found that there were, as expected, some fluctuations over time. However, when they examined the orders from the reseller, they observed much bigger swings. Also, to their surprise, they discovered that the orders from the printer division to the company's integrated circuit division had even greater fluctuations.

What happens when a supply chain is plagued with a bullwhip effect that distorts its demand information as it is transmitted up the chain? In the past, without being able to see the sales of its products at the distribution channel stage, HP had to rely on the sales orders from the resellers to make product forecasts, plan capacity, control inventory, and schedule production. Big variations in demand were a major problem for HP's management. The common symptoms of such variations could be excessive inventory, poor product forecasts, insufficient or excessive capacities, poor customer service due to unavailable products or long backlogs, uncertain production planning (i.e., excessive revisions), and high costs for corrections, such as for expedited shipments and overtime. HP's product division was a victim of order swings that were exaggerated by the resellers relative to their sales; it, in turn, created additional exaggerations of order swings to suppliers.

In the past few years, the Efficient Consumer Response (ECR) initiative has tried to redefine how the grocery supply chain should work.[1] One motivation for the initiative was the excessive amount of inventory in the supply chain. Various industry studies found that the total supply chain, from when products leave the manufacturers' production lines to when they arrive on the retailers' shelves, has more than 100 days of

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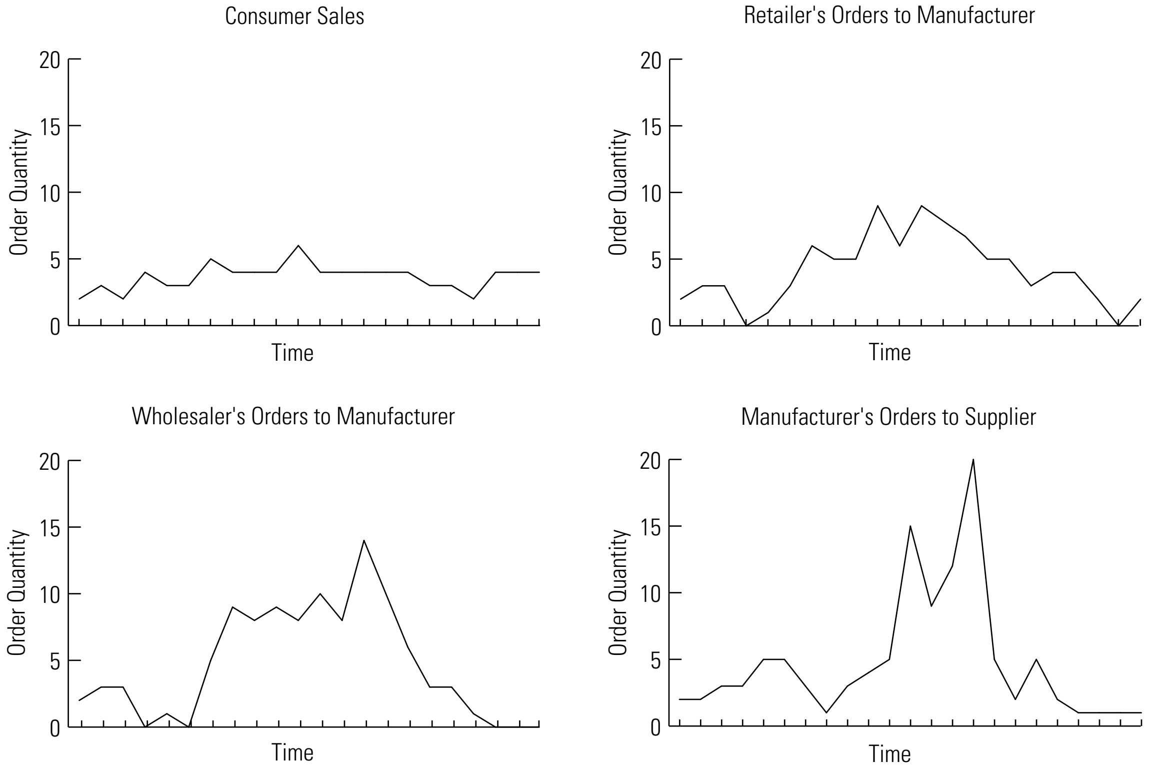
Figure 1 Increasing Variability of Orders up the Supply Chain



inventory supply. Distorted information has led every entity in the supply chain — the plant warehouse, a manufacturer's shuttle warehouse, a manufacturer's market warehouse, a distributor's central warehouse, the distributor's regional warehouses, and the retail store's storage space — to stockpile because of the high degree of demand uncertainties and variabilities. It's no wonder that the ECR reports estimated a potential $30 billion opportunity from streamlining the inefficiencies of the grocery supply chain.[2]

> The ordering patterns share a common, recurring theme: the variabilities of an upstream site are always greater than those of the downstream site.

Other industries are in a similar position. Computer factories and manufacturers' distribution centers, the distributors' warehouses, and store warehouses along the distribution channel have inventory stockpiles. And in the pharmaceutical industry, there are duplicated inventories in a supply chain of manufacturers such as Eli Lilly or Bristol-Myers Squibb, distributors such as McKesson, and retailers such as Longs Drug Stores. Again, information distortion can cause the total inventory in this supply chain to exceed 100 days of supply. With inventories of raw materials, such as integrated circuits and printed circuit boards in the computer industry and antibodies and vial manufacturing in the pharmaceutical industry, the total chain may contain more than one year's supply.

In a supply chain for a typical consumer product, even when consumer sales do not seem to vary much, there is pronounced variability in the retailers' orders to the wholesalers (see Figure 1). Orders to the manufacturer and to the manufacturers' supplier spike even more. To solve the problem of distorted information, companies need to first understand what creates the bullwhip effect so they can counteract it. Innovative companies in different industries have found that they can control the bullwhip effect and improve their supply chain performance by coordinating information and planning along the supply chain.

Causes of the Bullwhip Effect

Perhaps the best illustration of the bullwhip effect is the well-known "beer game."[3] In the game, participants (students, managers, analysts, and so on) play the roles of customers, retailers, wholesalers, and suppliers of a popular brand of beer. The participants cannot communicate with each other and must make order decisions based only on orders from the next downstream player. The ordering patterns share a common, recurring theme: the variabilities of an upstream site are always greater than those of the downstream site, a simple, yet powerful illustration of the bullwhip effect. This amplified order variability may be attributed to the players' irrational decision making. Indeed, Sterman's experiments showed that human behavior, such as misconceptions about inventory and demand information, may cause the bullwhip effect.[4]

In contrast, we show that the bullwhip effect is a consequence of the players' rational behavior within the supply chain's infrastructure. This important distinction implies that companies wanting to control the bullwhip effect have to focus on modifying the chain's infrastructure and related processes rather than the decision makers' behavior.

We have identified four major causes of the bullwhip effect:

1. Demand forecast updating
2. Order batching
3. Price fluctuation
4. Rationing and shortage gaming

Each of the four forces in concert with the chain's infrastructure and the order managers' rational decision making create the bullwhip effect. Understanding the causes helps managers design and develop strategies to counter it.[5]

Demand Forecast Updating

Every company in a supply chain usually does product forecasting for its production scheduling, capacity planning, inventory control, and material requirements planning. Forecasting is often based on the order history from the company's immediate customers.

The outcomes of the beer game are the consequence of many behavioral factors, such as the players' perceptions and mistrust. An important factor is each player's thought process in projecting the demand pattern based on what he or she observes. When a downstream operation places an order, the upstream manager processes that piece of information as a signal about future product demand. Based on this signal, the upstream manager readjusts his or her demand forecasts and, in turn, the orders placed with the suppliers of the upstream operation. We contend that demand signal processing is a major contributor to the bullwhip effect.

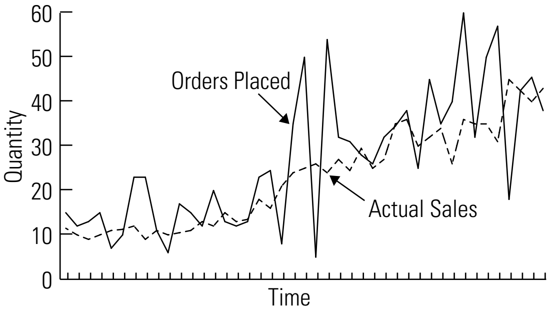
For example, if you are a manager who has to determine how much to order from a supplier, you use a simple method to do demand forecasting, such as exponential smoothing. With exponential smoothing, future demands are continuously updated as the new daily demand data become available. The order you send to the supplier reflects the amount you need to replenish the stocks to meet the requirements of future demands, as well as the necessary safety stocks. The future demands and the associated safety stocks are updated using the smoothing technique. With long lead times, it is not uncommon to have weeks of safety stocks. The result is that the fluctuations in the order quantities over time can be much greater than those in the demand data.

Now, one site up the supply chain, if you are the manager of the supplier, the daily orders from the manager of the previous site constitute your demand. If you are also using exponential smoothing to update your forecasts and safety stocks, the orders that you place with your supplier will have even bigger swings. For an example of such fluctuations in demand, see Figure 2. As we can see from the figure, the orders placed by the dealer to the manufacturer have much greater variability than the consumer demands. Because the amount of safety stock contributes to the bullwhip effect, it is intuitive that, when the lead times between the resupply of the items along the supply chain are longer, the fluctuation is even more significant.

Order Batching

In a supply chain, each company places orders with an upstream organization using some inventory monitoring or control. Demands come in, depleting inven-

Figure 2 Higher Variability in Orders from Dealer to Manufacturer than Actual Sales



tory, but the company may not immediately place an order with its supplier. It often batches or accumulates demands before issuing an order. There are two forms of order batching: periodic ordering and push ordering.

Instead of ordering frequently, companies may order weekly, biweekly, or even monthly. There are many common reasons for an inventory system based on order cycles. Often the supplier cannot handle frequent order processing because the time and cost of processing an order can be substantial. P&G estimated that, because of the many manual interventions needed in its order, billing, and shipment systems, each invoice to its customers cost between $35 and $75 to process.[6] Many manufacturers place purchase orders with suppliers when they run their material requirements planning (MRP) systems. MRP systems are often run monthly, resulting in monthly ordering with suppliers. A company with slow-moving items may prefer to order on a regular cyclical basis because there may not be enough items consumed to warrant resupply if it orders more frequently.

Consider a company that orders once a month from its supplier. The supplier faces a highly erratic stream of orders. There is a spike in demand at one time during the month, followed by no demands for the rest of the month. Of course, this variability is higher than the demands the company itself faces. Periodic ordering amplifies variability and contributes to the bullwhip effect.

One common obstacle for a company that wants to order frequently is the economics of transportation. There are substantial differences between full truckload (FTL) and less-than-truckload rates, so companies have a strong incentive to fill a truckload when they order materials from a supplier. Sometimes, suppliers give their best pricing for FTL orders. For most items, a full truckload could be a supply of a month or more. Full or close to full truckload ordering would thus lead to moderate to excessively long order cycles.

In push ordering, a company experiences regular surges in demand. The company has orders "pushed" on it from customers periodically because salespeople are regularly measured, sometimes quarterly or annually, which causes end-of-quarter or end-of-year order surges. Salespersons who need to fill sales quotas may "borrow" ahead and sign orders prematurely. The U.S. Navy's study of recruiter productivity found surges in the number of recruits by the recruiters on a periodic cycle that coincided with their evaluation cycle.[7] For companies, the ordering pattern from their customers is more erratic than the consumption patterns that their customers experience. The "hockey stick" phenomenon is quite prevalent.

When a company faces periodic ordering by its customers, the bullwhip effect results. If all customers' order cycles were spread out evenly throughout the week, the bullwhip effect would be minimal. The periodic surges in demand by some customers would be insignificant because not all would be ordering at the same time. Unfortunately, such an ideal situation rarely exists. Orders are more likely to be randomly spread out or, worse, to overlap. When order cycles overlap, most customers that order periodically do so at the same time. As a result, the surge in demand is even more pronounced, and the variability from the bullwhip effect is at its highest.

> Although some companies claim to thrive on high-low buying practices, most suffer.

If the majority of companies that do MRP or distribution requirement planning (DRP) to generate purchase orders do so at the beginning of the month (or end of the month), order cycles overlap. Periodic execution of MRPs contributes to the bullwhip effect, or "MRP jitters" or "DRP jitters."

Price Fluctuation

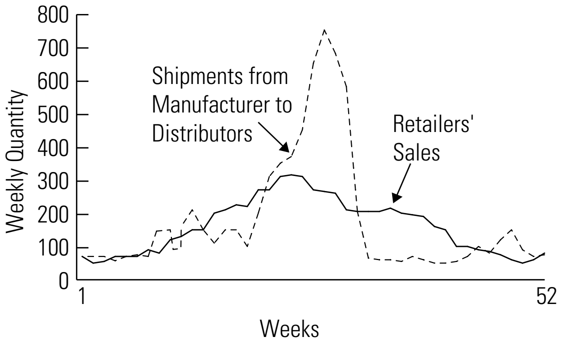
Estimates indicate that 80 percent of the transactions between manufacturers and distributors in the grocery industry were made in a "forward buy" arrangement in which items were bought in advance of requirements, usually because of a manufacturer's attractive price offer.[8] Forward buying constitutes $75 billion to $100 billion of inventory in the grocery industry.[9]

Forward buying results from price fluctuations in the marketplace. Manufacturers and distributors periodically have special promotions like price discounts, quantity discounts, coupons, rebates, and so on. All these promotions result in price fluctuations. Additionally, manufacturers offer trade deals (e.g., special discounts, price terms, and payment terms) to the distributors and wholesalers, which are an indirect form of price discounts. For example, Kotler reports that trade deals and consumer promotion constitute 47 percent and 28 percent, respectively, of their total promotion budgets.[10] The result is that customers buy in quantities that do not reflect their immediate needs; they buy in bigger quantities and stock up for the future.

Such promotions can be costly to the supply chain.[11] What happens if forward buying becomes the norm? When a product's price is low (through direct discount or promotional schemes), a customer buys in bigger quantities than needed. When the product's price returns to normal, the customer stops buying until it has depleted its inventory. As a result, the customer's buying pattern does not reflect its consumption pattern, and the variation of the buying quantities is much bigger than the variation of the consumption rate — the bullwhip effect.

When high-low pricing occurs, forward buying may well be a rational decision. If the cost of holding inventory is less than the price differential, buying in advance makes sense. In fact, the high-low pricing phenomenon has induced a stream of research on how companies should order optimally to take advantage of the low price opportunities.

Figure 3 Bullwhip Effect due to Seasonal Sales of Soup



Although some companies claim to thrive on high-low buying practices, most suffer. For example, a soup manufacturer's leading brand has seasonal sales, with higher sales in the winter (see Figure 3). However, the shipment quantities from the manufacturer to the distributors, reflecting orders from the distributors to the manufacturer, varied more widely. When faced with such wide swings, companies often have to run their factories overtime at certain times and be idle at others. Alternatively, companies may have to build huge piles of inventory to anticipate big swings in demand. With a surge in shipments, they may also have to pay premium freight rates to transport products. Damage also increases from handling larger than normal volumes and stocking inventories for long periods. The irony is that these variations are induced by price fluctuations that the manufacturers and the distributors set up themselves. It's no wonder that such a practice was called "the dumbest marketing ploy ever."[12]

Using trade promotions can backfire because of the impact on the manufacturers' stock performance. A group of shareholders sued Bristol-Myers Squibb when its stock plummeted from $74 to $67 as a result of a disappointing quarterly sales performance; its actual sales increase was only 5 percent instead of the anticipated 13 percent. The sluggish sales increase was reportedly due to the company's trade deals in a previous quarter that flooded the distribution channel with forward-buy inventories of its product.[13]

Rationing and Shortage Gaming

When product demand exceeds supply, a manufacturer often rations its product to customers. In one scheme, the manufacturer allocates the amount in proportion to the amount ordered. For example, if the total supply is only 50 percent of the total demand, all customers receive 50 percent of what they order. Knowing that the manufacturer will ration when the product is in short supply, customers exaggerate their real needs when they order. Later, when demand cools, orders will suddenly disappear and cancellations pour in. This seeming overreaction by customers anticipating shortages results when organizations and individuals make sound, rational economic decisions and "game" the potential rationing.[14] The effect of "gaming" is that customers' orders give the supplier little information on the product's real demand, a particularly vexing problem for manufacturers in a product's early stages. The gaming practice is very common. In the 1980s, on several occasions, the computer industry perceived a shortage of DRAM chips. Orders shot up, not because of an increase in consumption, but because of anticipation. Customers place duplicate orders with multiple suppliers and buy from the first one that can deliver, then cancel all other duplicate orders.[15]

More recently, Hewlett-Packard could not meet the demand for its LaserJet III printer and rationed the product. Orders surged, but HP managers could not discern whether the orders genuinely reflected real market demands or were simply phantom orders from resellers trying to get better allocation of the product. When HP lifted its constraints on resupply of the LaserJets, many resellers canceled their orders. HP's costs in excess inventory after the allocation period and in unnecessary capacity increases were in the millions of dollars.[16]

During the Christmas shopping seasons in 1992 and 1993, Motorola could not meet consumer demand for handsets and cellular phones, forcing many distributors to turn away business. Distributors like AirTouch Communications and the Baby Bells, anticipating the possibility of shortages and acting defensively, drastically overordered toward the end of 1994.[17] Because of such overzealous ordering by retail distributors, Motorola reported record fourth-quarter earnings in January 1995. Once Wall Street realized that the dealers were swamped with inventory and new orders for phones were not as healthy before, Motorola's stock tumbled almost 10 percent.

In October 1994, IBM's new Aptiva personal computer was selling extremely well, leading resellers to speculate that IBM might run out of the product before the Christmas season. According to some analysts, IBM, hampered by an overstock problem the previous year, planned production too conservatively. Other analysts referred to the possibility of rationing: "Retailers — apparently convinced Aptiva will sell well and afraid of being left with insufficient stock to meet holiday season demand — increased their orders with IBM, believing they wouldn't get all they asked for."[18] It was unclear to IBM how much of the increase in orders was genuine market demand and how much was due to resellers placing phantom orders when IBM had to ration the product.

How to Counteract the Bullwhip Effect

Understanding the causes of the bullwhip effect can help managers find strategies to mitigate it. Indeed, many companies have begun to implement innovative programs that partially address the effect. Next we examine how companies tackle each of the four causes. We categorize the various initiatives and other possible remedies based on the underlying coordination mechanism, namely, information sharing, channel alignment, and operational efficiency. With information sharing, demand information at a downstream site is transmitted upstream in a timely fashion. Channel alignment is the coordination of pricing, transportation, inventory planning, and ownership between the upstream and downstream sites in a supply chain. Operational efficiency refers to activities that improve performance, such as reduced costs and lead time. We use this topology to discuss ways to control the bullwhip effect (see Table 1).

Avoid Multiple Demand Forecast Updates

Ordinarily, every member of a supply chain conducts some sort of forecasting in connection with its planning (e.g., the manufacturer does the production planning, the wholesaler, the logistics planning, and so on). Bullwhip effects are created when supply chain members process the demand input from their immediate downstream member in producing their own forecasts. Demand input from the immediate downstream member, of course, results from that member's forecasting, with input from its own downstream member.

One remedy to the repetitive processing of consumption data in a supply chain is to make demand data at a downstream site available to the upstream site. Hence, both sites can update their forecasts with the same raw data. In the computer industry, manufacturers request sell-through data on withdrawn stocks from their resellers' central warehouse. Although the data are not as complete as point-of-sale (POS) data from the resellers' stores, they offer significantly more information than was available when manufacturers didn't know what happened after they shipped their products. IBM, HP, and Apple all require sell-through data as part of their contract with resellers.

Supply chain partners can use electronic data interchange (EDI) to share data. In the consumer products industry, 20 percent of orders by retailers of consumer products was transmitted via EDI in 1990.[19] In 1992, that figure was close to 40 percent and, in 1995, nearly 60 percent. The increasing use of EDI will undoubtedly facilitate information transmission and sharing among chain members.

Table 1 A Framework for Supply Chain Coordination Initiatives

Causes of Bullwhip	Information Sharing	Channel Alignment	Operational Efficiency
Demand Forecast Update	• Understanding system dynamics • Use point-of-sale (POS) data • Electronic data interchange (EDI) • Internet • Computer-assisted ordering (CAO)	• Vendor-managed inventory (VMI) • Discount for information sharing • Consumer direct	• Lead-time reduction • Echelon-based inventory control
Order Batching	• EDI • Internet ordering	• Discount for truckload assortment • Delivery appointments • Consolidation • Logistics outsourcing	• Reduction in fixed cost of ordering by EDI or electronic commerce • CAO
Price Fluctuations		• Continuous replenishment program (CRP) • Everyday low cost (EDLC)	• Everyday low price (EDLP) • Activity-based costing (ABC)
Shortage Gaming	• Sharing sales, capacity, and inventory data	• Allocation based on past sales	

Even if the multiple organizations in a supply chain use the same source demand data to perform forecast updates, the differences in forecasting methods and buying practices can still lead to unnecessary fluctuations in the order data placed with the upstream site. In a more radical approach, the upstream site could control resupply from upstream to downstream. The upstream site would have access to the demand and inventory information at the downstream site and update the necessary forecasts and resupply for the downstream site. The downstream site, in turn, would become a passive partner in the supply chain. For example, in the consumer products industry, this practice is known as vendor-managed inventory (VMI) or a continuous replenishment program (CRP). Many companies such as Campbell Soup, M&M/Mars, Nestlé, Quaker Oats, Nabisco, P&G, and Scott Paper use CRP with some or most of their customers. Inventory reductions of up to 25 percent are common in these alliances. P&G uses VMI in its diaper supply chain, starting with its supplier, 3M, and its customer, Wal-Mart. Even in the high-technology sector, companies such as Texas Instruments, HP, Motorola, and Apple use VMI with some of their suppliers and, in some cases, with their customers.

Inventory researchers have long recognized that multi-echelon inventory systems can operate better when inventory and demand information from downstream sites is available upstream. Echelon inventory — the total inventory at its upstream and downstream sites — is key to optimal inventory control.[20]

Another approach is to try to get demand information about the downstream site by bypassing it. Apple Computer has a "consumer direct" program, i.e., it sells directly to consumers without going through the reseller and distribution channel. A benefit of the program is that it allows Apple to see the demand patterns for its products. Dell Computers also sells its products directly to consumers without going through the distribution channel.

Finally, as we noted before, long resupply lead times can aggravate the bullwhip effect. Improvements in operational efficiency can help reduce the highly variable demand due to multiple forecast updates. Hence, just-in-time replenishment is an effective way to mitigate the effect.

Break Order Batches

Since order batching contributes to the bullwhip effect, companies need to devise strategies that lead to smaller batches or more frequent resupply. In addition, the counterstrategies we described earlier are useful. When an upstream company receives consumption data on a fixed, periodic schedule from its downstream customers, it will not be surprised by an unusually large batched order when there is a demand surge.

One reason that order batches are large or order frequencies low is the relatively high cost of placing an order and replenishing it. EDI can reduce the cost of the paperwork in generating an order. Using EDI, companies such as Nabisco perform paperless, computer-assisted ordering (CAO), and, consequently, customers order more frequently. McKesson's Economost ordering system uses EDI to lower the transaction costs from orders by drugstores and other retailers.[21] P&G has introduced standardized ordering terms across all business units to simplify the process and dramatically cut the number of invoices.[22] And General Electric is electronically matching buyers and suppliers throughout the company. It expects to purchase at least $1 billion in materials through its internally developed Trading Process Network. A paper purchase order that typically cost $50 to process is now $5.[23]

Another reason for large order batches is the cost of transportation. The differences in the costs of full truckloads and less-than-truckloads are so great that companies find it economical to order full truckloads, even though this leads to infrequent replenishments from the supplier. In fact, even if orders are made with little effort and low cost through EDI, the improvements in order efficiency are wasted due to the full-truckload constraint. Now some manufacturers induce their distributors to order assortments of different products. Hence a truckload may contain different products from the same manufacturer (either a plant warehouse site or a manufacturer's market warehouse) instead of a full load of the same product. The effect is that, for each product, the order frequency is much higher, the frequency of deliveries to the distributors remains unchanged, and the transportation efficiency is preserved. P&G has given discounts to distributors that are willing to order mixed-SKU (stock-keeping unit) loads of any of its products.[24] Manufacturers could also prepare and ship mixed SKUs to the distributors' warehouses that are ready to deliver to the stores.

"Composite distribution" for fresh produce and chilled products uses the same mixed-SKU concept to make resupply more frequent. Since fresh produce and chilled foods need to be stored at different temperatures, trucks to transport them need to have various temperatures. British retailers like Tesco and Sainsbury use trucks with separate compartments at different temperatures so that they can transport many products on the same truck.[25]

The use of third-party logistics companies also helps make small batch replenishments economical.[26] These companies allow economies of scale that were not feasible in a single supplier-customer relationship. By consolidating loads from multiple suppliers located near each other, a company can realize full truckload economies without the batches coming from the same supplier. Of course, there are additional handling and administrative costs for such consolidations or multiple pickups, but the savings often outweigh the costs.

> The simplest way to control the bullwhip effect caused by forward buying and diversions is to reduce both the frequency and the level of wholesale price discounting.

Similarly, a third-party logistics company can utilize a truckload to deliver to customers who may be competitors, such as neighboring supermarkets. If each customer is supplied separately via full truckloads, using third-party logistics companies can mean moving from weekly to daily replenishments. For small customers whose volumes do not justify frequent full truckload replenishments independently, this is especially appealing. Some grocery wholesalers that receive FTL shipments from manufacturers and then ship mixed loads to wholesalers' independent stores use logistics companies. In the United Kingdom, Sainsbury and Tesco have long used National Freight Company for logistics. As a result of the heightened awareness due to the ECR initiative in the grocery industry, we expect to see third-party logistics companies that forecast orders, transport goods, and replenish stores with mixed-SKU pallets from the manufacturers.

When customers spread their periodic orders or replenishments evenly over time, they can reduce the negative effect of batching. Some manufacturers coordinate their resupply with their customers. For example, P&G coordinates regular delivery appointments with its customers. Hence, it spreads the replenishments to all the retailers evenly over a week.

Stabilize Prices

The simplest way to control the bullwhip effect caused by forward buying and diversions is to reduce both the frequency and the level of wholesale price discounting. The manufacturer can reduce the incentives for retail forward buying by establishing a uniform wholesale pricing policy. In the grocery industry, major manufacturers such as P&G, Kraft, and Pillsbury have moved to an everyday low price (EDLP) or value pricing strategy. During the past three years, P&G has reduced its list prices by 12 percent to 24 percent and aggressively slashed the promotions it offers to trade customers. In 1994, P&G reported its highest profit margins in twenty-one years and showed increases in market share.[27] Similarly, retailers and distributors can aggressively negotiate with their suppliers to give them everyday low cost (EDLC). From 1991 to 1994, the percentage of trade deals in the total promotion budget of grocery products dropped from 50 percent to 47 percent.

From an operational perspective, practices such as CRP together with a rationalized wholesale pricing policy can help to control retailers' tactics, such as diversion. Manufacturers' use of CAO for sending orders also minimizes the possibility of such a practice.

Activity-based costing (ABC) systems enable companies to recognize the excessive costs of forward buying and diversions. When companies run regional promotions, some retailers buy in bulk in the area where the promotions are held, then divert the products to other regions for consumption. The costs of such practices are huge but may not show up in conventional accounting systems. ABC systems provide explicit accounting of the costs of inventory, storage, special handling, premium transportation, and so on that previously were hidden and often outweigh the benefits of promotions. ABC therefore helps companies implement the EDLP strategy.[28]

Eliminate Gaming in Shortage Situations

When a supplier faces a shortage, instead of allocating products based on orders, it can allocate in proportion to past sales records. Customers then have no incentive to exaggerate their orders. General Motors has long used this method of allocation in cases of short supply, and other companies, such as Texas Instruments and Hewlett-Packard, are switching to it.

"Gaming" during shortages peaks when customers have little information on the manufacturers' supply situation. The sharing of capacity and inventory information helps to alleviate customers' anxiety and, consequently, lessen their need to engage in gaming. But sharing capacity information is insufficient when there is a genuine shortage. Some manufacturers work with customers to place orders well in advance of the sales season. Thus they can adjust production capacity or scheduling with better knowledge of product demand.

Finally, the generous return policies that manufacturers offer retailers aggravate gaming. Without a penalty, retailers will continue to exaggerate their needs and cancel orders. Not surprisingly, some computer manufacturers are beginning to enforce more stringent cancellation policies.

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We contend that the bullwhip effect results from rational decision making by members in the supply chain. Companies can effectively counteract the effect by thoroughly understanding its underlying causes. Industry leaders like Procter & Gamble are implementing innovative strategies that pose new challenges: integrating new information systems, defining new organizational relationships, and implementing new incentive and measurement systems. The choice for companies is clear: either let the bullwhip effect paralyze you or find a way to conquer it. ◆

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CIPS- Bullwhip Effect In Supply Chain

https://www.cips.org/knowledge/procurement-topics-and-skills/operations-management/bullwhip-effect-in-supply-chain/

What is the bullwhip effect?

The bullwhip effect (also known as the Forrester effect) is defined as the demand distortion that travels upstream in the supply chain from the retailer through to the wholesaler and manufacturer due to the variance of orders which may be larger than that of sales.

What causes the bullwhip effect in supply chain?

- **Demand forecast updating:** Members of the supply chain updating their demand forecasting
- **Order batching:** Members of the supply chain rounding up or down the quantity of orders
- **Price fluctuations:** Usually driven by discounting resulting in larger quantities of purchases
- **Rationing and gaming:** Buyers and sellers delivering over or under their order quantities

An example of the bullwhip effect

Let's consider a retailer sells on average 10 ice creams per day in the summer season. Following a heatwave the retailer's sales increase to 30 units per day, in order to meet this new demand, the retailer increases their demand forecast and places an increased order on the wholesaler to 40 units per day in order to meet the new customer demand levels and to buffer any potential further increase in demand, this creates the first wave in the exaggerated demand being driven down the supply chain.

The wholesaler noticing this increase in demand from the retailer may then also build an incremental increase into their forecast so generating a larger order on the ice cream manufacturer, rather than ordering 40 units to be manufactured, the wholesaler may order 60 units from the manufacturer, this will further exaggerate the demand down the supply chain and so creates a second wave of demand increase.

The manufacturer also feeling the increase in demand from the wholesalers may also react to the increase by increasing their manufacturing run to 80 units, this creates a third wave in the exaggeration of demand.

The retailer may run out of stock during the heatwave whilst the manufacturer is producing new stock and may take the option of switching to an alternative brand to meet customer demand, this will then create a false demand situation as sales appear to slump to next to nothing so the retailer may then not place further demand for the original ice cream brand even though the manufacturer has increased their production runs. Alternatively, if the weather changes and the end consumers slow down on purchasing ice creams, this could result in an overstock situation across the supply chain as each tier of the supply chain has reacted to the heatwave sales and increased their demand. This is an example of the waves and troughs in the bullwhip effect.

How can the supply chain reduce the bullwhip effect?

The bullwhip effect in the supply chain can be reduced through shared knowledge with suppliers and customers. If members of the supply chain can determine what information is causing the overreactions this can be resolved. Communications and response times can be improved using modern technology.

The bullwhip effect can also be mitigated through these areas:

- Reduced lead times
- Revision of reordering procedures/better forecasting methods
- Limitations of price fluctuations
- Integration of planning and performance measurement

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The bullwhip effect in supply chains—An overestimated problem?

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ABSTRACT

A phenomenon that is now well known as the bullwhip effect suggests that the variability of orders increases as they move up the supply chain from retailers to wholesalers to manufacturers to suppliers. In this paper, we will focus mainly on measuring the bullwhip effect. Existing approaches that aim at quantifying the bullwhip effect neglect the network structure of supply chains. By only assuming a simple two-stage supply chain consisting of a single retailer and a single manufacturer, some of the relevant risk pooling effects associated with the network structure of supply chains are disregarded. Risk pooling effects arise when the orders, which a retailer receives from its customers, are statistically correlated with a coefficient of correlation less than one. When analyzing the bullwhip effect in supply chains, however, the influence of risk pooling has to be considered. The fact that these influences have not yet been analyzed motivates the research presented in this paper. We will show that the bullwhip effect is overestimated if just a simple supply chain is assumed and risk pooling effects are present.

1. Introduction

A phenomenon that is now well known as the bullwhip effect suggests that the variability of orders increases as we move upstream in the supply chain from retail to manufacturing. In a supply chain, although consumer sales do not seem to vary much, there is pronounced variability in the retailer's orders to the wholesaler. Furthermore, the wholesaler's order quantities to the manufacturer as well as the manufacturer's orders to the supplier vary even more in time. For a detailed elaboration of the bullwhip effect, see Kahn (1987), Lee et al. (1997a, b), and Metters (1997).

Numerous studies find the bullwhip effect in some industries and in numerous examples from individual products and companies. In the supply chain for diapers, *Procter and Gamble* (P&G) noticed that the volatility of the diaper orders issued by the distributors was quite high even though end consumer demand was reasonably stable (Lee et al., 1997b). In another paper, the same authors, Lee et al. (1997a), observe the bullwhip effect in a soup supply chain as well as in the supply chain for printers of *Hewlett–Packard* (HP). *Barilla* also finds that phenomenon in the supply chain for pasta (Hammond, 1994). Furthermore, Terwiesch et al. (2005) have found that the semiconductor equipment industry is more volatile than the personal computer industry, and Anderson et al. (2000) assign the volatility in the machine tool industry to the bullwhip effect. Additionally, the bullwhip effect has been experienced by many subjects playing *The Beer Game* (Sterman, 1989).

Regarding the large number of studies, which observed an increase in demand variability as one moves up a supply chain, Lee et al. (2004) conclude that nowadays "the bullwhip effect is a standard industry term and reference to it in industry publications has become commonplace" (p. 1891). In a seminal paper, these same authors identify four major causes of the bullwhip effect—(i) the updating of demand forecasts, (ii) order batching, (iii) price fluctuation, and (iv) rationing and shortage gaming—and suggest several managerial

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practices to mitigate its consequences. In addition, Dejonckheere et al. (2003) find that an important factor to the bullwhip effect is the replenishment rule used by the supply chain members. The authors conclude that whatever forecasting method is used (e.g. exponential smoothing or moving averages), order-up-to systems will always result in the bullwhip effect.

However, in a recent study of US industry level data, Cachon et al. (2005) find that demand volatility does not increase as one moves up the supply chain. In contrast to the natural consequences of the bullwhip effect, the authors observe that—in general—manufacturers do not have substantially greater demand volatility than retailers and may have even lower demand volatility. These results are explained mainly by production smoothing: predictable seasonality in combination with increasing marginal costs provides a strong motivation to smooth production relative to demand. Therefore, "the majority of retail and manufacturing industries smooth their production relative to their demand, i.e., they impose less volatility on their suppliers than they face from their customers" (Cachon et al., 2005, pp. 18–19). Cachon et al. (2005) conclude that the bullwhip effect is not widespread in the US economy and, moreover, the bullwhip effect is not commonplace.

In this paper, we analyze another strong force that mitigates the bullwhip effect. We focus on analyzing and measuring the bullwhip effect analytically. Because "supply chains" are more like "supply networks", our analysis accounts for supply chains that possess a network structure. In practice, supply chains can be considered as networks of geographically dispersed facilities—where raw materials, intermediate and finished products are produced, tested, modified, and stored—and the transportation links that connect the facilities. The different operations (e.g. raw materials procurement, finished goods manufacturing, and distribution) are performed on different stages of the supply chain. The term supply chain implies that only one player is involved at each stage of the supply chain. In reality, however, a manufacturer supplies several wholesalers and may receive material from several suppliers. Therefore, in the following, we use the term supply chain if only one player is involved at each stage, i.e. if the supply chain has a linear structure. If two or more players are involved at one stage, we will consequently use the term supply network.

Existing approaches that aim at quantifying the bullwhip effect neglect the network structure of supply chains. By assuming only a three-stage supply chain consisting of a single customer, a single retailer, and a single manufacturer, some relevant risk pooling effects associated with the network structure of supply chains are disregarded. Risk pooling effects arise for example when the orders a retailer receives from its customers are statistically correlated with a coefficient of correlation less than one. Note that the risk pooling effect is a special case of the well-known portfolio effect (Ronen, 1990). When analyzing the bullwhip effect in supply chains, however, the influence of risk pooling cannot be neglected. The fact that these influences have not been analyzed yet motivates the research presented in this paper.

We extend the analysis of Chen et al. (1999, 2000) to a supply chain with a network structure in which risk pooling can reduce the bullwhip effect on every individual stage. We first describe the supply chain setting, the forecasting technique, and the inventory policy used by the individual actors. To measure the bullwhip effect, the variances of the orders placed by the wholesalers to the manufacturers relative to the variance of the demand faced by the wholesalers will be determined. We will show that the bullwhip effect may be overestimated if just a simple supply chain is assumed and risk pooling effects are present. Therefore, in a supply network, using a simple forecasting method (e.g. moving averages), order-up-to systems will not always result in the bullwhip effect. The analytical results will be illustrated and affirmed by a simulation study.

2. Related literature

Since the first analysis of this phenomenon by Forrester (1958, 1961), the bullwhip effect has been addressed in a large number of publications. Recent research on the bullwhip effect can be divided into six general categories: (i) papers aiming at a quantification of the bullwhip effect (e.g. Carlsson and Fullér, 2000; Chen et al., 2000; Dejonckheere et al., 2003; Kahn, 1987; Lee et al., 1997a, b; Metters, 1997; Zhou and Disney, 2006), (ii) works focusing on analyzing and identifying the causes of the bullwhip effect (e.g. Geary et al., 2006; Lee et al., 1997a, b; Metters, 1997; Nienhaus et al., 2006), (iii) studies observing the bullwhip effect in some industries or in numerous examples from individual products and companies (e.g. Cachon et al., 2005; Lee et al., 1997a), (iv) papers addressing methods for reducing the bullwhip effect (e.g. Carlsson and Fullér, 2001; Chen et al., 1999; Dejonckheere et al., 2003; Disney and Towill, 2003; Ingalls et al., 2005; Mason-Jones and Towill, 2000; Moyaux et al., 2007), (v) works focusing on simulating the system behavior (e.g. Disney and Towill, 2003; Ingalls et al., 2005; Makajic-Nikolic et al., 2004; Nienhaus et al., 2006), and (vi) papers focusing on experimental validation of the bullwhip effect (e.g. Moyaux et al., 2003).

A great part of previous research has focused on demonstrating the existence and identifying the possible causes of the bullwhip effect (category (ii) of the relevant literature). Particularly, Lee et al. (1997a, b) identify four major causes of the considered phenomenon: (a) the updating of demand forecasts, (b) order batching, (c) price fluctuation, and (d) rationing and shortage gaming. The first cause occurs when the parties involved in the supply chain base their forecasts on the historical demand behavior of their immediate customers. Every supply chain member then adjusts to fluctuations of their order entry. Moreover, if every member reacts to fluctuations with smoothing techniques, the fluctuations will amplify throughout the supply chain. The effect of order batching—which is a rational order policy if the costs for frequent order processing are high—is an amplification of the order variability; the connection between the order policy and the actual demand patterns of the customers is then imparted. In case of price fluctuations customers are driven to buy in larger quantities by attractive offers, which may also include quantity discounts or price discounts. The resulting buying patterns will not reflect consumption patterns anymore, i.e. customers buy in quantities, which do not reflect their needs. Finally, the rationing and shortage game occurs when demand exceeds supply. The customers may start to exaggerate in comparison to their actual needs when there is a fear that supply will not cover demand (Carlsson and Fullér, 2000). These four causes are interdependent; the causes may interact and act in concert. However, the updating of demand forecasts appears to be the major source of the bullwhip effect.

The bullwhip effect has a number of negative effects in real supply chains, which can cause significant inefficiencies. The bullwhip effect typically leads to excessive inventory investments throughout the supply chain as the parties involved need to protect themselves against demand variations. Therefore, in another class of papers, methods reducing the bullwhip effect are proposed. While Lee et al. (1997b) suggest several managerial practices to reduce the bullwhip effect (e.g. the centralization of demand information), other papers specialized on using forecasting methods to reduce its consequences (e.g. Carlsson and Fullér, 2000; Dejonckheere et al., 2003). Carlsson and Fullér (2000) suggest a fuzzy approach to estimate future demand.

The category (i) of papers analyzing the bullwhip effect is focused on quantifying the increase in variability at each stage of the supply chain (e.g. Chen et al., 2000; Lee et al., 1997b; Metters, 1997). However, these approaches neglect the more complex network structure of real supply chains. By assuming only a simple two-stage supply chain consisting of a single retailer and a single manufacturer, some relevant risk pooling effects associated with the network structure of supply chains—therefore often called supply networks—are disregarded. Risk pooling effects arise, for example, when the orders a retailer receives from its customers are statistically correlated with a coefficient of correlation less than one. When analyzing the bullwhip effect in supply networks, the influence of risk pooling has to be considered. In the following, we will show that the bullwhip effect is overvalued if just a simple supply chain is assumed and risk pooling effects in supply networks are present.

3. The bullwhip effect in supply chains

The analysis of the bullwhip effect will be based on the approaches of Chen et al. (1999, 2000), Metters (1997), Lee et al. (1997b), and Kahn (1987), which consider a two-stage supply chain. In this paper, we will analyze a three-stage supply chain consisting—at first—of a single retailer, a single wholesaler, and a single manufacturer (see Fig. 1).

3.1. The inventory policy

The considered inventory context is the following. We assume an inventory system managed by the wholesaler with periodic review, where D_t is the stochastic and stationary demand of the retailer in any period t. By stationary, we imply that the retailer's demand in different time periods fluctuates randomly around a constant mean level. We assume that the retailer's demands are independent over time and identically distributed random variables. Assuming that a review is made at the start of each period $t \in \{1,2,\ldots,T\}$, under an order-up-to policy the wholesaler places an order q_t to the manufacturer considering the target inventory level y_t for period t. Afterwards, the wholesaler fills the retailer's demand D_t for period t from on-hand inventory. Note that any unfilled demands—the shortages—are backlogged (except at the end of period T, when they are lost). In order to focus on the influence of risk pooling on the bullwhip effect, we set, in contrast to Chen et al. (2000), the lead time to zero, which means that any order placed at the start of period t is also available at the beginning of period t. This is how we can quantify the bullwhip effect without the influence of lead times.

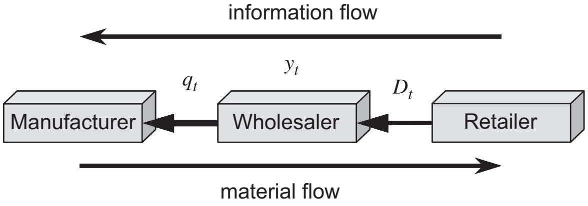

Fig. 1. The three-stage supply chain.

In order to determine the orders q_t, similar to Chen et al. (2000), we assume that the wholesaler follows an *order-up-to inventory policy*. The goal of this ordering policy is to bring the actual inventory towards the desired inventory y_t (Johnson and Montgomery, 1974). The order quantity q_t, the wholesaler places to the manufacturer at the start of period t, is given by

$$q_t = y_t - y_{t-1} + D_{t-1}, \tag{1}$$

where y_t is the desired order-up-to level (target inventory), y_{t-1} the order-up-to level at the end of period $t-1$, and D_{t-1} the perceived demand. In case of $q_t<0$ we assume that this *excess inventory* is returned to the manufacturer without cost, i.e. we allow costless returns (Kahn, 1987; Lee et al., 1997b). According to Chen et al. (2000), we assume that the wholesaler follows a simple *order-up-to inventory policy* in which the order-up-to level (target stock level) in period t is estimated from the observed demand as

$$y_t = E(D_t) + z\sqrt{\mathrm{Var}(D_t)}, \tag{2}$$

where $E(D_t)$ is an estimate of the mean demand, $\sqrt{\mathrm{Var}(D_t)}$ an estimate of the standard deviation of the retailer's demand, and $z \geqslant 0$ is the safety factor chosen to meet a desired service level (Silver et al., 1998). Note that z is a managerial determined factor that indicates the number of estimated standard deviations of demand to be kept as safety stock (Zinn et al., 1989). For $z=0$ the decision maker is risk neutral, and for $z>0$ the decision maker is risk averse.

3.2. The forecasting technique

To estimate $E(D_t)$ and $\mathrm{Var}(D_t)$—following Chen et al. (2000)—the wholesaler uses the simple N-period moving average. The estimated mean of the retailer's demand in period t is given by

$$E(D_t) = \left(\frac{1}{N}\right)\sum_{i=t-N}^{t-1} D_i = \left(\frac{1}{N}\right)(D_{t-1} + D_{t-2} + \cdots + D_{t-N}),$$

i.e. the mean of the N most recent observations is used as the forecast for the next period. We will use the notation MA(N) for N-period moving averages. The estimated variance of the retailer's demand in t is given by

$$\mathrm{Var}(D_t) = \left(\frac{1}{N}\right)\sum_{i=t-N}^{t-1}(D_i - E(D_t))^2 = \left(\frac{1}{N}\right)((D_{t-1} - E(D_t))^2 + \cdots + (D_{t-N} - E(D_t))^2).$$

Note that the estimated values of $E(D_t)$ and $\mathrm{Var}(D_t)$ may change every period and, hence, the wholesaler's order-up-to level changes also every period (Simchi-Levi et al., 2000). Given the estimates of the mean demand $E(D_t)$ and the standard deviation $\sqrt{\mathrm{Var}(D_t)}$ we can write the order quantity q_t as

$$\begin{aligned} q_t &= E(D_t) + z\sqrt{\mathrm{Var}(D_t)} - E(D_{t-1}) - z\sqrt{\mathrm{Var}(D_{t-1})} \\ &\quad + D_{t-1} = \left(\frac{1}{N}\right)\left(\sum_{i=t-N}^{t-1} D_i - \sum_{i=t-1-N}^{t-2} D_i\right) \\ &\quad + z\left(\sqrt{\mathrm{Var}(D_t)} - \sqrt{\mathrm{Var}(D_{t-1})}\right) + D_{t-1} \\ &= \left(\frac{1}{N}\right)(D_{t-1} - D_{t-1-N}) + z\left(\sqrt{\mathrm{Var}(D_t)}\right. \\ &\quad \left. - \sqrt{\mathrm{Var}(D_{t-1})}\right) + D_{t-1} \\ &= \left(1 + \frac{1}{N}\right)D_{t-1} + \left(-\frac{1}{N}\right) \\ &\quad D_{t-1-N} + z\left(\sqrt{\mathrm{Var}(D_t)} - \sqrt{\mathrm{Var}(D_{t-1})}\right). \end{aligned} \tag{3}$$

The following illustrative example shows the determination of the orders placed by the wholesaler depending on the periodical demands of the retailer. In this example, we use a 2-period moving average MA(2) and a safety factor (z value) of $z = 2.33$ (representing the desired service level). The calculation was performed on a Microsoft Excel® spreadsheet (Table 1).

3.3. Quantifying the bullwhip effect in supply chains

To quantify the bullwhip effect Chen et al. (2000) suggest using $\mathrm{Var}(q)/\mathrm{Var}(D)$, where $\mathrm{Var}(D)$ denotes the variance of the retailer's demand and $\mathrm{Var}(q)$ refers to the variance of orders placed by the wholesaler. In the example presented above, regarding the order quantities $q_3, \ldots, q_{20}$ and the demands $D_3, \ldots, D_{20}$, the resulting bullwhip effect (for the regarded periods) is given by $\mathrm{Var}(q)/\mathrm{Var}(D) = 2266.63/265.27 = 8.54$ (see Fig. 2).

In order to analytically quantify the increase in variability from the wholesaler to the manufacturer, i.e. to quantify the bullwhip effect, we first determine the variance of the orders placed by the wholesaler to the manufacturer. Using the forecast method presented in Section 3.2, the order quantities q_t are given by (3). Based on (3) the estimated variance of the wholesaler's order quantity in t is given by

$$\begin{aligned} \mathrm{Var}(q_t) &= \left(1 + \frac{1}{N}\right)^2 \mathrm{Var}(D_{t-1}) + \left(-\frac{1}{N}\right)^2 \mathrm{Var}(D_{t-1-N}) \\ &\quad + z^2 \mathrm{Var}\left(\sqrt{\mathrm{Var}(D_t)} - \sqrt{\mathrm{Var}(D_{t-1})}\right) \\ &\quad - 2\left(1 + \frac{1}{N}\right)\frac{1}{N}\mathrm{Cov}(D_{t-1}, D_{t-1-N}) \\ &\quad + 2z\left(1 + \frac{2}{N}\right)\mathrm{Cov}\left(D_{t-1}, \sqrt{\mathrm{Var}(D_t)}\right). \end{aligned} \tag{4}$$

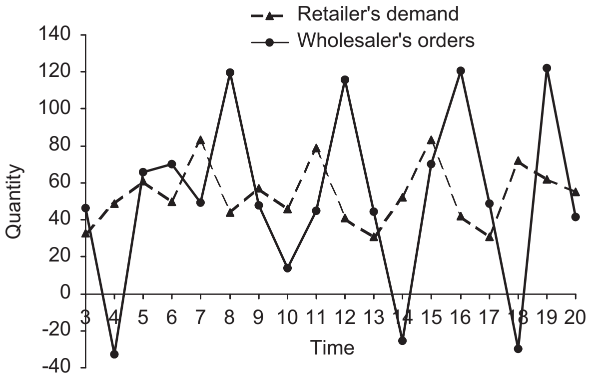

Fig. 2. Illustration of the bullwhip effect.

Table 1
Determination of wholesaler's order quantities

Period	0	1	2	3	4	5	6	7	8	9	10
D_t	50	81	39	33	49	61	50	83	44	57	46
$E(D_t)$			65.5	60	36	41	55	55.5	66.5	63.5	50.5
$\mathrm{Var}(D_t)$			240.25	441	9	64	36	30.25	272.25	380.25	42.25
y_t			101.62	108.93	42.99	59.64	68.98	68.32	104.95	108.94	65.65
q_t				46.32	−32.94	65.65	70.34	49.34	119.63	47.99	13.71
Period		11	12	13	14	15	16	17	18	19	20
D_t		79	41	31	52	83	42	31	72	62	55
$E(D_t)$		51.5	62.5	60	36	41.5	67.5	62.5	36.5	51.5	67
$\mathrm{Var}(D_t)$		30.25	272.25	361	25	110.25	240.25	420.25	30.25	420.25	25
y_t		64.32	100.95	104.27	47.65	65.97	103.62	110.27	49.32	99.27	78.65
q_t		44.67	115.63	44.33	−25.62	70.32	120.65	48.65	-29.95	121.95	41.39

Note that the estimated values of $\mathrm{Var}(q_t)$ and $\mathrm{Var}(D_t)$ may change every period. However, the bullwhip effect is measured by the quotient $\mathrm{Var}(q)/\mathrm{Var}(D)$, i.e. in order to quantify the bullwhip effect we have to determine the variances of the wholesaler's order quantity and the retailer's demand for the complete planning period. Considering the planning period [1,T] with the order quantities $q_1, \ldots, q_T$ and the realized demands $D_1, \ldots, D_T$, the sample variances and are given by $\mathrm{Var}(q) = (1/T)\sum_{t=1}^{T}(q_t - E(q))^2$, with $E(q) = (1/T)\sum_{t=1}^{T} q_t$, and $\mathrm{Var}(D) = (1/T)\sum_{t=1}^{T}(D_t - E(D))^2$, with $E(D) = (1/T)\sum_{t=1}^{T} D_t$.

Regarding the specific structure of the estimated variance of the wholesaler's order quantity at each period $t \in \{1, 2, \ldots, T\}$—as shown in (4)—a lower bound on the variance for the complete planning period can be calculated (see Chen et al. (2000), p. 438). Chen et al. (2000) show that

$$\mathrm{Cov}\left(D_{t-i}, \sqrt{\mathrm{Var}(D_t)}\right) = 0 \quad \forall \quad i = 1, \ldots, N.$$

Assuming that the retailer's demands are stochastically independent, i.e. the demands are independent and identically distributed, follows $\mathrm{Cov}(D_{t-1}, D_{t-1-N}) = 0$. In order to calculate a lower bound on the wholesaler's orders' variance, we furthermore assume $z = 0$. So, we are able to determine a lower bound on the variance of the wholesaler's order quantity for the whole planning period as

$$\begin{aligned}\mathrm{Var}(q) &= \left(1 + \frac{1}{N}\right)^2 \mathrm{Var}(D) + \left(-\frac{1}{N}\right)^2 \mathrm{Var}(D)\\ &= \left(1 + \frac{2}{N} + \frac{2}{N^2}\right)\mathrm{Var}(D). \qquad (5)\end{aligned}$$

Furthermore, we are able to determine a lower bound on the increase in variability from the wholesaler to the manufacturer, i.e. the bullwhip effect, as (see Chen et al. (2000), pp. 438–439)

$$\frac{\mathrm{Var}(q)}{\mathrm{Var}(D)} \geqslant \left(1 + \frac{2}{N} + \frac{2}{N^2}\right). \qquad (6)$$

The lower bound (6) is tight for $z = 0$, i.e. if the wholesaler's target stock level is solely depending on the retailer's mean demand. Note that Chen et al. (2000) exhibit that the lower bound describes the behavior of the system accurately even in cases where $z \neq 0$. The relationship (6) shows that the increase in variability is a decreasing function of N, the number of observations N used to estimate the mean and the variance of the retailer's demand.

4. The bullwhip effect in supply networks

Chen et al. (2000) pointed out that the approach presented above does not capture many of the complexities involved in real world supply chains. In the following, we will extend this approach to account for the typical network structure of real supply chains. It will be shown that the bullwhip effect may be overestimated if just a simple supply chain is assumed.

In practice, supply chains often exhibit a network structure comprising of geographically dispersed production facilities, warehouses, and transportation links connecting the aforementioned locations. The supply chain can be subdivided into different stages where different operations (e.g. raw materials procurement, finished goods manufacturing, and distribution) are performed. In general, the operations performed at each stage of the supply chain are distributed among several geographically dispersed facilities owned by different companies. The number of stages, the number of facilities at each stage, and the number of links between the locations determine the network structure of the supply chain and consequently also the material flow from the raw materials stage to the final customer stage.

Extending the approach presented above, we assume a three-stage supply network (see Fig. 3) consisting of two retailers, a single wholesaler, and a single manufacturer. We refer to the first retailer as party ($R1$) and use subscript "$R1$" to designate his set of parameters. Similarly, we refer to the second retailer in our supply network as party ($R2$), and, hence generally use subscript "$R2$" to designate his set of parameters. In each period t, the wholesaler places an order q_t to the manufacturer. Afterwards the wholesaler fills the demands for that period, denoted by $D_{R1,t}$ for retailer ($R1$) and $D_{R2,t}$ for retailer ($R2$). The overall demand for period t is given by $\sum_{j=1}^{J} D_{Rj,t}$, with $J = 2$. As in the previous section, we assume that the wholesaler follows a simple *order-up-to inventory policy* (see Eqs. (1), (2), and (3)).

4.1. The risk pooling effect in supply networks

To motivate our further analysis, we first show a simple statistical effect. If the wholesaler calculates the order-up-to level for the retailers' demands in an isolated manner it follows $\bar{y}_{R1,t} = E(D_{R1,t}) + z\sqrt{\mathrm{Var}(D_{R1,t})}$ and $\bar{y}_{R2,t} = E(D_{R2,t}) + z\sqrt{\mathrm{Var}(D_{R2,t})}$. In this case, the overall order-up-to level in period t is given by

$$\begin{aligned}\bar{y}_t = \bar{y}_{R1,t} + \bar{y}_{R2,t} &= E(D_{R1,t}) + E(D_{R2,t})\\ &\quad + z\left(\sqrt{\mathrm{Var}(D_{R1,t})} + \sqrt{\mathrm{Var}(D_{R2,t})}\right). \qquad (7)\end{aligned}$$

Note that—because of the zero lead time assumption—the overall order-up-to level in (7) corresponds to the inventory level (= stock on-hand—backorders). However, when the wholesaler calculates the order-up-to level based on the aggregated demand, $\sum_{j=1}^{J} D_{Rj,t}$, the wholesaler's order-up-to level (target stock level) in period

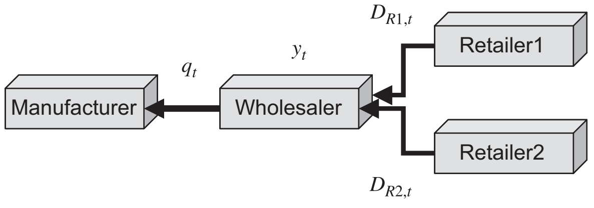

Fig. 3. The three-stage supply network.

t is given by

$$\begin{aligned}\tilde{y}_t &= E\left(\sum_{j=1}^{2} D_{Rj,t}\right) + z\sqrt{\mathrm{Var}\left(\sum_{j=1}^{2} D_{Rj,t}\right)}\\ &= E\left(\sum_{j=1}^{2} D_{Rj,t}\right)\\ &\qquad + z\sqrt{\mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t}) + 2\mathrm{Cov}(D_{R1,t}, D_{R2,t})},\end{aligned} \tag{8}$$

where $\mathrm{Cov}(D_{R1,t},D_{R2,t})$ is the covariance between the demand of retailer ($R1$) and the demand of retailer ($R2$). If the retailers' demands are stochastically independent, the covariance is zero. Then, the order-up-to level in (8) for the aggregated demand is given by

$$\tilde{y}_t = E(D_{R1,t}) + E(D_{R2,t}) + z\sqrt{\mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t})}$$

It is easy to see that the total $(\sqrt{\mathrm{Var}(D_{R1,t})} + \sqrt{\mathrm{Var}(D_{R2,t})})$ in (7) is greater than $\sqrt{\mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t})}$:

$$\begin{aligned}&\sqrt{\mathrm{Var}(D_{R1,t})} + \sqrt{\mathrm{Var}(D_{R2,t})} > \sqrt{\mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t})}\\ &\quad \Leftrightarrow \mathrm{Var}(D_{R1,t}) + 2\sqrt{\mathrm{Var}(D_{R1,t})}\sqrt{\mathrm{Var}(D_{R2,t})}\\ &\quad + \mathrm{Var}(D_{R2,t}) > \mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t})\\ &\quad \Leftrightarrow 2\sqrt{\mathrm{Var}(D_{R1,t})}\sqrt{\mathrm{Var}(D_{R2,t})} > 0.\end{aligned} \tag{9}$$

Thus, the order-up-to levels $\tilde{y}_t$ based on the aggregated demand are less than the overall order-up-to levels $\bar{y}_t = \bar{y}_{R1,t} + \bar{y}_{R2,t}$ based on an isolated manner.

In the following, the retailers' demands $D_{R1,t}$ and $D_{R2,t}$ are statistically correlated with a coefficient of correlation $-1 < \rho_D < 1$. With

$$\rho_D = \frac{\mathrm{Cov}(D_{R1,t}, D_{R2,t})}{\sqrt{\mathrm{Var}(D_{R1,t})}\sqrt{\mathrm{Var}(D_{R2,t})}},$$

the order-up-to level $\tilde{y}_t$ in (8) can be rewritten as

$$\begin{aligned}\tilde{y}_t &= E\left(\sum_{j=1}^{2} D_{Rj,t}\right)\\ &\quad + z\sqrt{\mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t}) + 2\rho_D\sqrt{\mathrm{Var}(D_{R1,t})}\sqrt{\mathrm{Var}(D_{R2,t})}}.\end{aligned} \tag{10}$$

Note that the retailers' demands are perfectly positively correlated if $\rho_D = 1$. The demands are perfectly negatively correlated if $\rho_D = -1$. If $\rho_D = 1$ the variance of the aggregated demand is given by

$$\begin{aligned}&\mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t}) + 2\sqrt{\mathrm{Var}(D_{R1,t})}\sqrt{\mathrm{Var}(D_{R2,t})}\\ &= \left(\sqrt{\mathrm{Var}(D_{R1,t})} + \sqrt{\mathrm{Var}(D_{R2,t})}\right)^2.\end{aligned}$$

For a correlation coefficient $\rho_D = -1$ the variance of the aggregated demand is given by

$$\begin{aligned}&\mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t}) - 2\sqrt{\mathrm{Var}(D_{R1,t})}\sqrt{\mathrm{Var}(D_{R2,t})}\\ &= \left(\sqrt{\mathrm{Var}(D_{R1,t})} - \sqrt{\mathrm{Var}(D_{R2,t})}\right)^2.\end{aligned}$$

With the correlation coefficient $\rho_D \in \{-1, 1\}$ the upper and lower bounds of the variance of the aggregated demand can be calculated as

$$\begin{aligned}&\left(\sqrt{\mathrm{Var}(D_{R1,t})} - \sqrt{\mathrm{Var}(D_{R2,t})}\right)^2\\ &\quad \leqslant \mathrm{Var}\left(\sum_{j=1}^{2} D_{Rj,t}\right) \leqslant \left(\sqrt{\mathrm{Var}(D_{R1,t})} + \sqrt{\mathrm{Var}(D_{R2,t})}\right)^2.\end{aligned} \tag{11}$$

The upper bound in (11) reveals a typical risk pooling effect in supply networks: in case of a coefficient of correlation $\rho_D < 1$ a lower safety stock level has to be held to satisfy the desired safety level. The resulting reduction of inventory is well known as *the square root law*, which was proven mathematically by Maister (1976). Mathematically stated, the square root law says that total inventory in a system is proportional to the square root of the number of warehouses in which a product is stocked; i.e. consolidating warehouses will have lower stock levels. Zinn et al. (1989), Zinn et al. (1990), and Ronen (1990) measure the effect of inventory centralization on stock level. Tyagi and Das (1998) analyze the structure of inventory systems based on the square root law. Lee et al. (1993) show that the effect resulting from the square root law is one reason for centralizing inventories in supply chains. Lee and Billington (1992, 1993, 1995) as well as Lee et al. (1993) demonstrate how HP utilized this risk pooling effect by implementing a so-called *design for localization strategy*. The risk pooling effect based on the square root law is a special case of the well-known portfolio effect.

4.2. The forecasting technique regarding risk pooling

The wholesaler still uses the simple N-period moving average MA(N):

$$\begin{aligned}E\left(\sum_{j=1}^{2} D_{Rj,t}\right) &= E(D_{R1,t}) + E(D_{R2,t})\\ &= \frac{1}{N}\left(\sum_{i=t-N}^{t-1} D_{R1,i} + \sum_{i=t-N}^{t-1} D_{R2,i}\right)\\ &= \frac{1}{N}\left(\sum_{i=t-N}^{t-1} D_{R1,i}\right) + \frac{1}{N}\left(\sum_{i=t-N}^{t-1} D_{R2,i}\right),\end{aligned} \tag{12}$$

$$\begin{aligned}\mathrm{Var}\left(\sum_{j=1}^{2} D_{Rj,t}\right) &= \mathrm{Var}(D_{R1,t}) + \mathrm{Var}(D_{R2,t})\\ &\quad + 2\rho_D\sqrt{\mathrm{Var}(D_{R1,t})}\sqrt{\mathrm{Var}(D_{R2,t})}\\ &= \frac{1}{N}\Bigg(\sum_{i=t-N}^{t-1}(D_{R1,i} - E(D_{R1,t}))^2\\ &\quad + \sum_{i=t-N}^{t-1}(D_{R2,i} - E(D_{R2,t}))^2\\ &\quad + 2\rho_D\sqrt{\sum_{i=t-N}^{t-1}(D_{R1,i} - E(D_{R1,t}))^2}\\ &\quad \sqrt{\sum_{i=t-N}^{t-1}(D_{R2,i} - E(D_{R2,t}))^2}\Bigg).\end{aligned} \tag{13}$$

The order quantity, $\tilde{q}_t$, the wholesaler places to the manufacturer in period t—regarding the risk pooling effect presented above—is given by $\tilde{q}_t = \tilde{y}_t - \tilde{y}_{t-1} + \sum_{j=1}^{2} D_{Rj,t}$. To quantify the increase in variability from the wholesaler to the manufacturer, we determine the variance of the orders placed by the wholesaler. In order to calculate a lower bound on the variance of the wholesaler's orders we, once again, assume $z = 0$:

$$\begin{aligned}\mathrm{Var}(\tilde{q}) &= \left(1 + \frac{2}{N} + \frac{2}{N^2}\right)\mathrm{Var}\left(\sum_{j=1}^{2} D_{Rj}\right)\\ &= \left(1 + \frac{2}{N} + \frac{2}{N^2}\right)\Big(\mathrm{Var}(D_{R1}) + \mathrm{Var}(D_{R2})\\ &\quad + 2\rho_D\sqrt{\mathrm{Var}(D_{R1})}\sqrt{\mathrm{Var}(D_{R2})}\Big). \qquad (14)\end{aligned}$$

If the wholesaler's calculation is based on isolated demands of both retailers, with Eq. (7) the demand's variance is given by $\mathrm{Var}(D_{R1}+)\mathrm{Var}(D_{R2}) + 2\sqrt{\mathrm{Var}(D_{R1})}\sqrt{\mathrm{Var}(D_{R2})}$. Therefore, if the wholesaler calculates the order quantities in an isolated manner, i.e. $\bar{q}_{R1,t} = \bar{y}_{R1,t} - \bar{y}_{R1,t-1} + D_{R1,t-1}$, $\bar{q}_{R2,t} = \bar{y}_{R2,t} - \bar{y}_{R2,t-1} + D_{R2,t-1}$ and $\bar{q}_t = \bar{q}_{R1,t} + \bar{q}_{R2,t}$, the lower bound on the variance of the wholesaler's overall orders is given by

$$\begin{aligned}\mathrm{Var}(\bar{q}) &= \left(1 + \frac{2}{N} + \frac{2}{N^2}\right)\Big(\mathrm{Var}(D_{R1}) + \mathrm{Var}(D_{R2})\\ &\quad + 2\sqrt{\mathrm{Var}(D_{R1})}\sqrt{\mathrm{Var}(D_{R2})}\Big).\end{aligned}$$

Consequently, if the wholesaler considers possible risk pooling effects while calculating the order quantity, in case of correlation $\rho_D < 1$ the orders' variance $\mathrm{Var}(\tilde{q})$ is less than the orders' variance $\mathrm{Var}(\bar{q})$, i.e. without considering risk pooling. However, if we now determine the lower bound on the increase in variability from the wholesaler to the manufacturer, i.e. the bullwhip effect, the relative increase in variability generally does not change and is still given by Eq. (6) as $\mathrm{Var}(q)/\mathrm{Var}(D) \geqslant (1 + (2/N) + (2/N^2))$. Yet, the total increase in variability within the supply network regarding risk pooling is less than without taking risk pooling into account. The absolute increase of the variances of the orders placed by the wholesaler relative to the variances of the demands is given by

$$\begin{aligned}\mathrm{Var}(\tilde{q}) - \mathrm{Var}\left(\sum_{j=1}^{2} D_{Rj}\right) &= \left(\frac{2}{N} + \frac{2}{N^2}\right)\mathrm{Var}\left(\sum_{j=1}^{2} D_{Rj}\right)\\ &= \left(\frac{2}{N} + \frac{2}{N^2}\right)\Big(\mathrm{Var}(D_{R1}) + \mathrm{Var}(D_{R2})\\ &\quad + 2\rho_D\sqrt{\mathrm{Var}(D_{R1})}\sqrt{\mathrm{Var}(D_{R2})}\Big). \qquad (15)\end{aligned}$$

Therefore, in case of $\rho_D < 1$ the absolute increase of the variances of the wholesaler's orders to the variances of the demands in supply networks is less than without consideration of risk pooling.

5. Experiments and simulation

5.1. An illustrative example

In the following, we will illustrate the reduction of the bullwhip effect if risk pooling effects are present. In this example, the wholesaler preserves orders of two retailers ($R1$) and ($R2$). In the initial situation, the wholesaler determines his orders separately based on the individual demand of each retailer, i.e. without consideration of a potential correlation between the retailers' demands. In this case, the overall order-up-to level in period t is given by $\bar{y}_t = \bar{y}_{R1,t} + \bar{y}_{R2,t}$ (see Eq. (7)). With $\bar{q}_t = \bar{q}_{R1,t} + \bar{q}_{R2,t}$ the orders of the wholesaler are calculated. The wholesaler uses a 2-period moving average MA(2) and a safety factor (z value) of $z = 2.33$ (representing the desired service level). The calculation was performed on a Microsoft Excel® spreadsheet. Table 2 shows the relevant data for retailer ($R1$), i.e. the data to calculate $\bar{y}_{R1,t} = E(D_{R1,t}) + z\sqrt{\mathrm{Var}(D_{R1,t})}$ and $\bar{q}_{R1,t} = \bar{y}_{R1,t} - \bar{y}_{R1,t-1} + D_{R1,t-1}$. Table 3 shows the relevant data for retailer ($R2$), which are used for $\bar{y}_{R2,t} = E(D_{R2,t}) + z\sqrt{\mathrm{Var}(D_{R2,t})}$ and $\bar{q}_{R2,t} = \bar{y}_{R2,t} - \bar{y}_{R2,t-1} + D_{R2,t-1}$. Table 4 shows $\bar{y}_t = \bar{y}_{R1,t} + \bar{y}_{R2,t}$ and $\bar{q}_t = \bar{q}_{R1,t} + \bar{q}_{R2,t}$.

Fig. 4 shows that the wholesaler's orders vary even more than the retailers' periodical demands. Calculating the sample variances $\mathrm{Var}(\bar{q})$ and $\mathrm{Var}(D)$ for the whole planning period—see Section 3.3—the bullwhip effect without consideration risk pooling is given by $\mathrm{Var}(\bar{q})/\mathrm{Var}(D) = 1997.22/197.69 = 10.1$ or $\mathrm{Var}(\bar{q}) - \mathrm{Var}(D) = 1799.53$.

Determining the order-up-to levels and the orders for the retailers' demands in an isolated manner neglects the correlation of the demands. Analyzing the retailers' periodical demands reveals that the demands of retailer ($R1$) and retailer ($R2$) are statistically correlated with a correlation coefficient of $\rho_D = -0.5$. Table 5 shows the relevant data and the results $\tilde{y}_t$ and $\tilde{q}_t$. The target stock level $\tilde{y}_t$ is calculated by using Eq. (10) and the order quantity results from $\tilde{q}_t = \tilde{y}_t - \tilde{y}_{t-1} + \sum_{j=1}^{2} D_{Rj,t}$. Note that the correlation coefficient ρ_D is defined for the whole time series. Therefore, computing the estimated variance of the retailers' demand $\mathrm{Var}(D_t)$ in period t, based on (13), may differ from the results, which are obtained using the technique in Section 3.2. However, Fig. 5 shows that the bullwhip effect is reduced significantly if the wholesaler considers the statistical correlation of the retailers' demands. Regarding the statistically correlated demands of the retailers, the bullwhip effect results to $\mathrm{Var}(\tilde{q})/\mathrm{Var}(D) = 400.24/197.69 = 2.02$ or $\mathrm{Var}(\tilde{q}) - \mathrm{Var}(D) = 202.55$.

5.2. A simulation study

Besides the illustrative example presented above, we have implemented a simulation study. Within this study,

Table 2
The wholesaler's orders depending on the demand of retailer ($R1$)

Period	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$D_{R1,t}$	94.46	103.7	81.69	135.37	99.63	98.05	86.82	100.40	104.32	89.33	84.70	105.68	120.92	91.21	109.74	90.75	104.08	119.80	107.21	123.18	72.90
$E(D_{R1,t})$			99.05	92.67	108.53	117.50	98.84	92.43	93.61	102.36	96.82	87.01	95.19	113.30	106.07	100.47	100.24	97.41	111.94	113.51	115.20
$\mathrm{Var}(D_{R1,t})$			21.12	120.52	720.40	319.29	0.63	31.55	46.13	3.84	56.17	5.37	110.08	58.07	220.62	85.78	90.09	44.37	61.84	39.64	63.77
$\bar{y}_{R1,t}$			109.76	118.25	171.07	159.14	100.69	105.52	109.43	106.93	114.29	92.41	119.63	131.05	140.67	122.05	122.36	112.93	130.26	128.18	133.80
$\bar{q}_{R1,t}$				90.18	188.19	87.70	39.60	91.65	104.31	101.81	96.69	62.82	132.90	132.34	100.83	91.12	91.06	94.65	137.13	105.13	128.81

Table 3
The wholesaler's orders depending on the demand of retailer ($R2$)

Period	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$D_{R2,t}$	99.08	89.55	123.32	72.48	72.69	92.61	107.71	76.74	90.49	104.21	87.74	99.04	104.23	83.22	89.31	116.53	102.40	90.59	90.79	78.75	128.18
$E(D_{R2,t})$			94.31	106.43	97.90	72.58	82.65	100.16	92.23	83.62	97.35	95.98	93.39	101.64	93.72	86.26	102.92	109.46	96.49	90.69	84.77
$Var(D_{R2,t})$			22.74	285.18	646.25	0.01	99.17	56.98	239.65	47.22	47.10	67.82	31.93	6.73	110.43	9.27	185.24	49.92	34.85	0.01	36.25
$\bar{y}_{R2,t}$			105.43	145.78	157.13	72.83	105.85	117.75	128.29	99.63	113.34	115.17	106.56	107.68	118.21	93.36	134.63	125.92	110.25	90.92	98.79
$\bar{q}_{R2,t}$				163.67	83.83	–11.60	125.63	119.60	87.29	61.82	117.93	89.57	90.43	105.36	93.74	64.45	157.80	93.69	74.91	71.46	86.62

Table 4
The wholesaler's orders without consideration risk pooling

Period	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$D_{R1,t}+D_{R2,t}$	207.85	172.33	190.66	194.52	177.14	194.81	193.54	172.44	204.72	225.15	174.43	199.04	207.28	206.47	210.39	198.00	201.93	201.08
$\bar{y}_t = \bar{y}_{R1,t} + \bar{y}_{R2,t}$	264.03	328.2	231.97	206.85	223.27	237.72	206.56	227.63	207.58	226.19	238.73	258.88	215.41	256.99	238.85	240.51	219.1	232.59
$\tilde{q}_t = \tilde{q}_{R1,t}+\tilde{q}_{R2,t}$	253.85	272.02	76.1	165.23	211.25	191.6	163.63	214.62	152.39	223.33	237.7	194.57	155.57	248.86	188.34	212.04	176.59	215.43

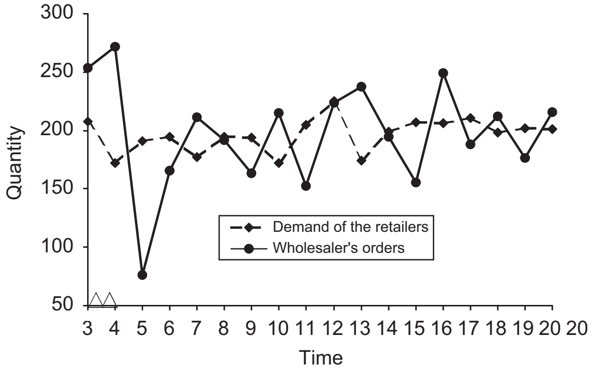

Fig. 4. Demand and order quantities without consideration of risk pooling.

we have simulated the demand for time series of 1000 periods, with $E(D_{R1}) = E(D_{R2}) = 100$ and $\mathrm{Var}(D_{R1}) = \mathrm{Var}(D_{R2}) = 400$ over all periods and a safety factor (z value) of $z = 1.645$ (representing the desired service level of 95%). The correlation coefficient varies between $\rho_D \in [-0.9;0.9]$ in steps of 0.1; i.e. we have analyzed 19 scenarios for the correlation coefficient. Each scenario was calculated 100 times.

Fig. 6 shows that the average bullwhip effect for each scenario varies between 2.5 and 3; i.e. $2.5 < Var(\tilde{q})/\mathrm{Var}(D) < 3.0$. As Chen et al. (2000) noted, the lower bound (6) accurately describes the behavior of the system even in cases where $z \neq 0$.

In our simulation study we use a 2-period moving average MA(2), i.e. $N = 2$. Therefore, the resulting lower bound is given by $\mathrm{Var}(\tilde{q})/\mathrm{Var}(D) \geqslant 2.5$. The relative increase in variability does not vary depending on the correlation coefficient. It is easy to see that the bullwhip effect exists definitely. However, Fig. 7 shows the variance of the demand Var(D) and the variance of the orders placed by the wholesaler Var(q) depending on the value of the correlation coefficient. It is shown that both the variance of the demand Var(D) and the variance of the orders placed by the wholesaler Var(q) decrease for lower values of the correlation coefficient. Measuring the bullwhip effect by the absolute difference Var(q)–Var(D), the reduction of the bullwhip effect for correlated demands is considerable (see Fig. 8).

6. The bullwhip effect in supply networks with multiple retailers

In the preceding section, we have simplified our analysis to the case of two retailers at one stage of the supply network. In reality, a wholesaler often supplies more than two retailers. If the wholesaler fills the demands of J retailers, with $J > 2$, the variance of the aggregated demand can be calculated as (see Mood

Table 5
The wholesaler's orders with consideration risk pooling

Period	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
D_t	193.5	193.2	205.01	207.85	172.33	190.66	194.52	177.14	194.81	193.54	172.44	204.72	225.15	174.43	199.04	207.28	206.47	210.39	198.00	201.93	201.08
$E(D_t)$			193.37	199.10	206.43	190.09	181.49	192.59	185.83	185.98	194.18	182.99	188.58	214.94	199.79	186.74	203.16	206.88	208.43	204.19	199.96
$\mathrm{Var}(D_t)$			0.03	34.92	2.01	315.47	84.02	3.73	75.5	77.99	0.39	111.36	260.57	104.35	643.22	151.45	16.96	0.16	3.84	38.41	3.86
$\tilde{y}_t$			204.28	233.69	267.38	231.60	203.83	208.42	217.15	200.26	210.95	200.13	209.77	230.57	230.61	205.79	231.33	222.89	224.95	218.75	216.76
$\tilde{q}_t$				234.41	241.54	136.54	162.89	199.11	185.88	177.92	204.23	161.62	214.37	245.95	174.47	174.22	232.82	198.03	212.45	191.80	199.93

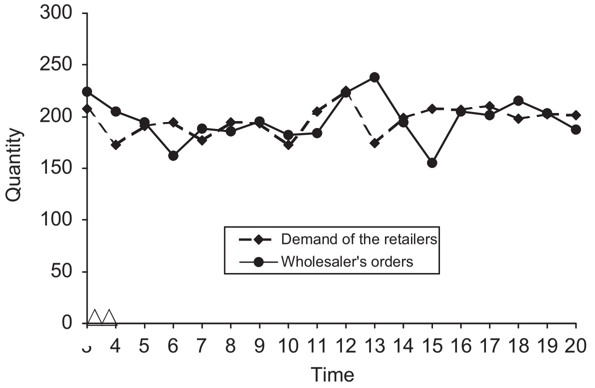

Fig. 5. Demand and order quantities with consideration of risk pooling.

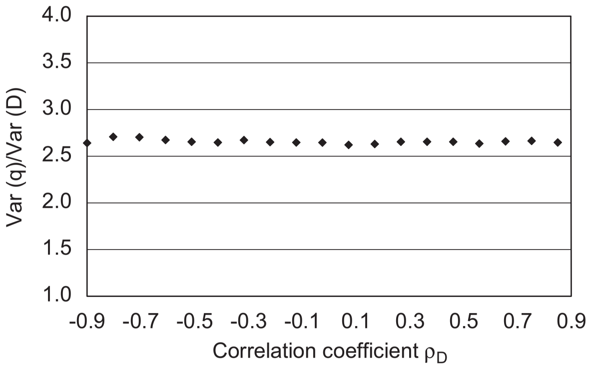

Fig. 6. Bullwhip effect depending on the value of the correlation coefficient.

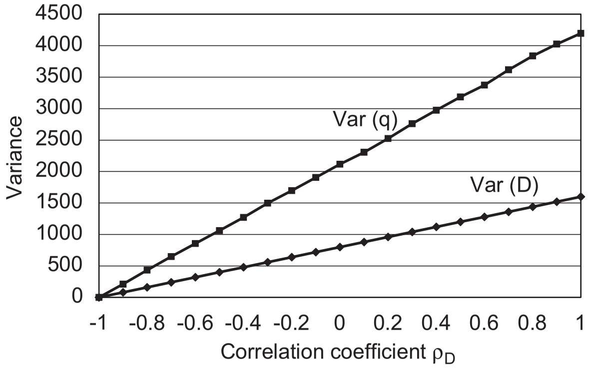

Fig. 7. Variances of the demands and the orders depending on the correlation coefficient.

et al., 1974)

$$\mathrm{Var}\left(\sum_{j=1}^{J} D_{Rj,t}\right) = \sum_{j=1}^{J}\mathrm{Var}(D_{Rj,t}) + 2\sum_{j=1}^{J}\sum_{k=j+1}^{K}\mathrm{Cov}(D_{Rj,t}, D_{Rk,t}). \tag{16}$$

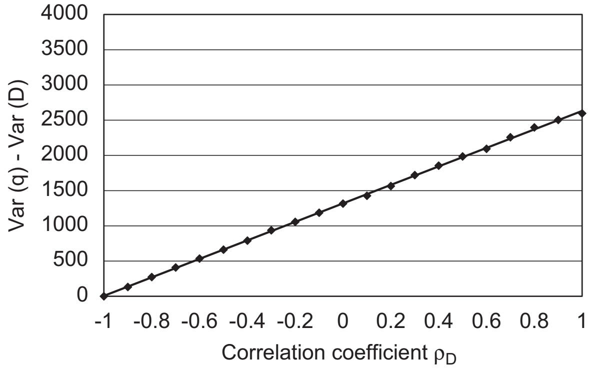

Fig. 8. The absolute difference Var(q)–Var(D) depending on the correlation coefficient.

The Eq. (16) can be expressed using the covariance matrix:

$$\begin{bmatrix} E[(D_{R1,t}-E(D_{R1,t}))(D_{R1,t}-E(D_{R1,t}))] & \cdots & E[(D_{R1,t}-E(D_{R1,t}))(D_{RJ,t}-E(D_{RJ,t}))] \\ E[(D_{R2,t}-E(D_{R1,t}))(D_{R1,t}-E(D_{R1,t}))] & \cdots & E[(D_{R2,t}-E(D_{R2,t}))(D_{RJ,t}-E(D_{RJ,t}))] \\ \vdots & \vdots & \vdots \\ E[(D_{RJ,t}-E(D_{RJ,t}))(D_{R1,t}-E(D_{R1,t}))] & \cdots & E[(D_{RJ,t}-E(D_{RJ,t}))(D_{RJ,t}-E(D_{RJ,t}))] \end{bmatrix}$$
$$= \begin{bmatrix} Var(D_{R1,t}) & Cov(D_{R1,t},D_{R2,t}) & \cdots & Cov(D_{R1,t},D_{RJ,t}) \\ \mathrm{Cov}(D_{R2,t},D_{R1,t}) & \mathrm{Var}(D_{R2,t}) & \cdots & \mathrm{Cov}(D_{R2,t},D_{RJ,t}) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Cov}(D_{RJ,t},D_{R1,t}) & \mathrm{Cov}(D_{RJ,t},D_{R2,t}) & \cdots & \mathrm{Var}(D_{RJ,t}) \end{bmatrix}. \qquad (17)$$

In case of J retailers ($J>2$), the wholesaler may still use the N-period moving average MA(N) determining $E(\sum_{j=1}^{J} D_{Rj,t})$ and $\mathrm{Var}(\sum_{j=1}^{J} D_{Rj,t})$:

$$E\left(\sum_{j=1}^{J} D_{Rj,t}\right) = \frac{1}{N}\left(\sum_{i=t-N}^{t-1} D_{R1,i} + \sum_{i=t-N}^{t-1} D_{R2,i} + \cdots + \sum_{i=t-N}^{t-1} D_{RJ,i}\right), \qquad (18)$$

$$\begin{aligned} \mathrm{Var}\left(\sum_{j=1}^{J} D_{Rj,t}\right) = \left(\frac{1}{N}\right)\Bigg(&\sum_{i=t-N}^{t-1}(D_{R1,i}-E(D_{R1,t}))^2 \\ &+ \sum_{i=t-N}^{t-1}(D_{R2,i}-E(D_{R2,t}))^2 + \cdots \\ &+ \sum_{i=t-N}^{t-1}(D_{RJ,i}-E(D_{RJ,t}))^2 \\ &+ 2\sum_{j=1}^{J}\sum_{k=j+1}^{K}\left(\rho_{j,k}\sqrt{\sum_{i=t-N}^{t-1}(D_{Rj,i}-E(D_{Rj,t}))^2} \right. \\ &\left.\sqrt{\sum_{i=t-N}^{t-1}(D_{Rk,i}-E(D_{Rk,t}))^2}\right)\Bigg). \end{aligned} \qquad (19)$$

With (18) and (19) the order quantity q_t, the wholesaler places to the manufacturer in each period t, can be calculated. Finally, using the difference Var(q)–Var(D), the bullwhip effect can be determined in case of $J>2$ retailers. Then, risk pooling effects resulting from correlation coefficients less than one could strongly reduce the bullwhip effect.

7. Conclusion

By assuming a three-stage supply chain consisting of a single retailer, a single wholesaler, and a single manufacturer relevant risk pooling effects associated with the network structure of supply chains are neglected. Based on the approaches by Chen et al. (1999, 2000) we have shown that the bullwhip effect may be overestimated if just a supply chain is assumed and risk pooling effects in supply networks can be utilized. If we take into the consideration that in practice forecasting methods superior to the simple N-period moving average are used and regarding the findings of Cachon et al. (2005), we can conclude that the bullwhip effect is present; however, the bullwhip effect is not commonplace. Finally, referring to Dejonckheere et al. (2003), we come to the logical conclusion that order-up-to systems usually result in the bullwhip effect, but the strength of the effect depends on the statistical correlation of the regarded demands.

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